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ADVISORY GROUP FOR AEROSPACE RESEARCH & DEVELOPMENT

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AGARDograph No. 162

on

Acoustic Fatigue Design Data

Part I

by

A.G.R.Thomson

NORTH ATLANTIC TREATY ORGANIZATION

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ADVISORY GROUP FOR AEROSPACE RESEARCH AND DEVELOPMENT

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AGARDograph 162

ACOUSTIC FATIGUE DESIGN DATA

Part I

by

A.G.R.Thomson

Engineering Sciences Data Unit Ltd
London, UK

This AGARDograph was sponsored by the Structures and Materials Panel of AGARD.

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PREFACE

~~This volume~~, the first part of a series giving data for design against acoustic fatigue, has been prepared in order to draw together the results of research in acoustic fatigue and to present them in a form directly useable in aerospace design. Future work in this series will deal with endurance of aluminium alloy and titanium alloy structures under simulated acoustic loading, stress response of flat or curved honeycomb panels, near field compressor noise estimation, stress response of box and control surface structures and structural damping.

The AGARD Structures and Materials Panel has for many years been active in encouraging and coordinating the work that has been necessary to make this collection of design data possible and after agreeing on procedures for the acquisition, analysis and interpretation of the requisite data, work on this series of design data sheets was initiated in 1970.

The overall management of the project has been conducted by the Working Group on Acoustic Fatigue of the AGARD Structures and Materials Panel, and the project has been financed through a collective fund established by the Nations collaborating in the project, namely Canada, France, Germany, Italy, U.K. and U.S. National Coordinators appointed by each country have provided the basic data, liaised with the sources of the data, and provided constructive comment on draft data sheets. These Coordinators are Dr G.M. Lindberg (Canada), Mr R. Loubet (France), Mr G. Bayerdörfer (Germany), Gen. A. Griselli (Italy), Mr N.A. Townsend (U.K.), Mr A.W. Kolb (U.S.) and Mr F.F. Rudder (U.S.). Staff of the Engineering Sciences Data Unit Ltd, London, have analysed the basic data and prepared and edited the resultant data sheets with invaluable guidance and advice from the National Coordinators and from the Acoustic Fatigue Panel of the Royal Aeronautical Society which has the following constitution: Professor B.L. Clarkson (Chairman), Mr D.C.G. Eaton, Mr J.A. Hay, Mr W.T. Kirkby, Mr M.J.T. Smith and Mr N.A. Townsend. The members of Staff of the Engineering Sciences Data Unit concerned with the preparation of the data sheets in this volume are: Mr A.G.R. Thomson (Executive, Environmental Projects), Dr G. Sen Gupta and Mr R.F. Lambert (Environmental Projects Group).

Data sheets based on this AGARDograph will subsequently be issued in the Fatigue Series of Engineering Sciences Data issued by ESDU Ltd, where additions and amendments will be made to maintain their current applicability.

AHHall

A.H. Hall

Chairman,
Working Group on Acoustic Fatigue
Structures and Materials Panel

CONTENTS

Section 1

INTRODUCTION TO DESIGN INFORMATION ON ACOUSTIC FATIGUE

1.1 General Remarks

Experience on many different aircraft has shown that the wide band high intensity noise associated with a jet exhaust can cause structural fatigue failure in regions close to the jet. Similar failures have occurred in other regions of pressure fluctuation such as close to propeller tips and in regions of separated flow, for example behind airbrakes. To date there is no known case of catastrophic failure of an aircraft due to acoustic fatigue but the damage to the structure can lead to unacceptable expense for maintenance, inspection and loss of aircraft use.

This Introduction outlines the important factors in acoustic fatigue failure and gives guidance on the use of data in this AGARDograph and in relevant Engineering Sciences Data Items. In the present state of knowledge a complete analytical solution can not be presented, but the framework of a design procedure applicable especially to skin panels can be described.

Terms used in acoustic fatigue analysis are defined in Reference 1.5.2 and a short bibliography of relevant literature is given in Reference 1.5.3.

1.2 Outline of the Problem

Table 1.1 summarises the important factors affecting the acoustic fatigue life of a structure subjected to jet noise. Similar factors apply for other sources of pressure fluctuation, when the relevant factors of importance for the operating conditions and noise field characteristics need to be considered in place of 1 and 2 in Table 1.1.

Table 1.1

	Principal Items	Factors of Importance
Loading Actions	1. Operating condition	Flight plan Time at full thrust and low forward speed Use of reverse thrust Use of afterburner Occurrence of shock cell noise
Loading Actions	2. Noise field characteristics	Engine characteristics Engine position Reflected noise from ground and other structure
3.	Structural response	Natural frequencies Mode shapes Modal damping
4.	Stress at critical points	Predominant modes Detail design (stress concentration)
5.	Fatigue damage	Long life portion of S_{rms}-N_r curve under random loading Crack propagation characteristics Environment (corrosion, temperature)

1.3 Design Procedure

1.3.1 Skin panels

The basic type of skin design for a component subject to acoustic fatigue loading is chosen with regard to the overall noise level which may be estimated as outlined in Section 1.4.2 or from previous experience. Guidance on the choice of skin design and on good design practice is given in Reference 1.5.4. The effect of variations in panel geometry may be investigated by means of the analysis outlined in Sections 1.4.1 to 1.4.4. Use of early design charts such as those of Reference 1.5.1 has been found to lead to excessively heavy designs in many cases, but the later References are more realistic.

Tests of critical sections of structure in a simulated jet noise environment may be necessary after preliminary design, and after redesign a proof test may be required before finalisation.

1.4 Analysis

The analytical estimation of the life of a part subject to acoustic fatigue requires information in three main areas:

(a) the loading action, i.e. the noise field characteristics and durations under critical operating conditions,

(b) the structural response, i.e. the natural frequencies of the structure and the amplitudes of the induced stresses,

(c) the fatigue life of the type of structure made from the particular material when subjected to the resultant random loading.

The following notes amplify these information requirements.

1.4.1 Operating conditions

The starting point for an analysis is the estimation of the pressure fluctuations and their durations throughout the flight plan or mission profile, and the estimated number of flights during service life. For example important elements may be the total time during which the engine is at full thrust and the forward speed is low, the possible occurrence of shock cell noise in cruise, and the use of reverse thrust in landing. In addition consideration should be given to pressure fluctuations such as those generated by jet exhaust impingement and boundary layer turbulence. Consideration of such elements leads to an estimate of the acoustic fatigue load spectrum.

Reference 1.5.11 contains a suggested procedure for dealing with cases where more than one operating condition is a significant source of acoustic fatigue damage.

Reference 1.5.6 deals with the combination of sound levels from two or more sources acting simultaneously.

1.4.2 Noise-field characteristics

The loading action on the structure is computed from a knowledge of the engine noise characteristics and the vehicle configuration. The most important factors are the location of the engines relative to the structure and the proximity of sound reflecting surfaces such as wing, fuselage, tail section or runway. The characteristics required to define the loading action completely are

(a) the pressure spectra and spatial distribution over the structural surface,

(b) the correlation of the pressures over the structural surface.

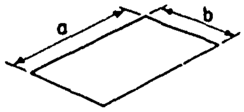
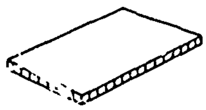
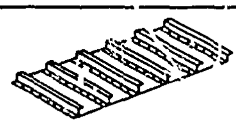
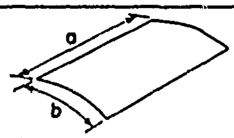
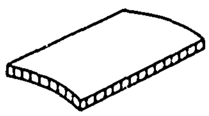
A method of estimating the first, in relation to jet noise, is given in Section 2 and further methods are listed in Reference 1.5.3. Although estimates by the method of Section 2 have compared well with measurements on several different full-scale and model aircraft, it usually remains necessary in the design development stage to supplement them by measurements of noise levels on prototype aircraft.

The spatial correlation of the pressures is not usually required in initial design because of the simplifying assumptions which must be made.

1.4.3 Structural response and stress at critical points

The complete expression for the response of a structure to acoustic excitation requires a knowledge of all the modes of vibration of the structure, their natural frequencies and damping, in addition to the noise field characteristics. In practice, the response of the structure can be estimated only from simplified representations of the structure and of the pressure field.

The following table shows sources of data on natural frequencies of various types of structure in their lower modes.

Type of Structure	Edge Conditions	Data Source
 Flat plate	All edges fixed All edges simply-supported	Ref. 1.5.8
 Flat honeycomb panel	All edges simply-supported	Ref 1 5.9 (to be revised in 1972)
 Stringer-stiffened panel	Torsional restraint at inflexible stringers, long edge of skin simply-supported, short edge restrained in rotation	Section 3.
 Curved plate	All edges fixed All edges simply-supported (a > b)	Section 4
 Curved honeycomb panel	All edges fixed All edges simply-supported	To be issued in Part 2, 1972

In calculating natural frequencies and stress response, experience suggests that individual plate widths should be taken as the distance between adjacent skin-stiffener rivet lines. For bonded structures, the distance between centrelines of adjacent bonded flanges is generally taken.

Knowing the natural frequencies, the sound pressure levels and the pressure spectra, and making assumptions on panel damping, Section 5 gives an approximate method of estimating the stress response, in terms of r.m.s. stress, of skin-stringer panels having stiffeners of relatively high flexural stiffness; Reference 1.12 outlines the simplified theory used and shows a comparison with experimental results. In using this method it is required to know the spectrum level of acoustic pressure at a natural frequency of the panel. If band levels only are known, they may be converted to spectrum level using Reference 1.5.5. The effect on the stresses of reinforcement by increased thickness along the plate edges is considered in Reference 1.5.13.

1.4.4 Fatigue strength and endurance

The estimated r.m.s. stress in the panel is compared with appropriate fatigue strength data in order to estimate the life of the panel. In making this comparison it is important to ensure that the fatigue strength data used correspond to the appropriate conditions because the highest stresses usually occur in the region of stiffener or frame rivet lines where effects of stress concentration and fretting are important. Reference 1.10 gives data on the endurance of various types of joint in aluminium alloys subjected to simulated acoustic loading.

Where a panel is subjected to acoustic loadings of different intensities at different times it is suggested tentatively that the effects should be combined by applying the simple cumulative damage hypothesis using S_{rms}-N_r data. This approach is discussed in Reference 1.5.11.

1.5 References

1.5.1 - Structural design for acoustic fatigue. WADC ASD-TDR-63-820, 1963.

1.5.2 - Definitions of terms for acoustic fatigue analysis. Engineering Sciences Data Item No. 66013, 1966.

1.5.3 - A short bibliography on acoustic fatigue.
Engineering Sciences Data Item No. 66014, 1966.

1.5.4 - General principles for the design of acoustically or similarly excited structure.
Engineering Sciences Data Item No. 66015, 1966.

1.5.5 - Bandwidth correction.
Engineering Sciences Data Item No. 66016, 1966.

1.5.6 - Combination of levels in dB.
Engineering Sciences Data Item No. 66017, 1966.

1.5.7 - The relation between sound pressure level and r.m.s. fluctuating pressure.
Engineering Sciences Data Item No. 66018, 1966.

1.5.8 - Natural frequencies of uniform flat plates.
Engineering Sciences Data Item No. 66019, 1966.

1.5.9 - Natural frequencies of flat sandwich panels with cores of zero flexural stiffness and simply-supported edges.
Engineering Sciences Data Item No. 66020, 1966.

1.5.10 - Endurance of riveted skin-rib flange connections (tentative). (Aluminium alloy material - In bending.)
Engineering Sciences Data Item No. 66022, 1966.
(To be superseded in Part 2 of this AGARDograph.)

1.5.11 Kirkby, W.T.
Edwards, P.R. A method of fatigue life prediction using data obtained under random loading conditions.
RAE tech. Rep. 66023, 1966.

1.5.12 - Estimation of the r.m.s. stress in skin panels subjected to random acoustic loading.
Engineering Sciences Data Item No. 67028, 1967.

1.5.13 - The effect of edge reinforcement on the stresses in skin panels under uniform pressure.
Engineering Sciences Data Item No. 67029, 1967.

1.5.14 - Near-field noise analysis of aircraft propulsion systems with emphasis on prediction techniques for jets.
AFFDL-TR-67-43, 1967.

1.5.15 - Refinement of sonic fatigue structural design criteria.
AFFDL-TR-67-156, 1968.

Section 2

THE ESTIMATION OF NEAR FIELD SOUND PRESSURE LEVELS DUE TO JET NOISE

2.1 Notation

c_p	specific heat at constant pressure of fully expanded jet gases	J/kg K	ft lbf/slug K
D	jet nozzle diameter	m	ft
f	frequency	Hz	c/s
L	overall sound pressure level	dB	dB
L_o	datum overall sound pressure level	dB	dB
L_v	correction to L due to change in jet velocity	dB	dB
L_ρ	correction to L due to change in jet density	dB	dB
n	velocity index		
p	ambient pressure	N/m^2	lbf/ft^2
R	gas constant (287 J/kg K) (3090 ft lbf/slug K)	J/kg K	ft lbf/slug K
T_j	jet pipe static temperature	K	K
T_{Hj}	jet pipe total temperature	K	K
V	mean fully expanded jet velocity	m/s	ft/s
V_o	datum fully expanded jet velocity (610 m/s) (2000 ft/s)	m/s	ft/s
x	axial distance from jet pipe nozzle plane, measured in jet direction	m	ft
y	radial distance from jet axis	m	ft
ρ_j	density of jet gases	kg/m^3	slug/ft^3
ρ_o	datum density (0.49 kg/m^3) (0.000 95 slug/ft^3)	kg/m^3	slug/ft^3

Both SI and British units are quoted but any coherent system of units may be used.

2.2 Introduction

This Section gives a method of estimating the near field sound pressure levels due to high velocity jet noise from a single stationary conical nozzle.

Detailed noise data from the engine manufacturer should be used for preference but, when this is not possible, a number of methods of estimation is available. These methods give widely differing results and after comparing them the following method has been found to give the closest agreement with experimental results. This method is intended to give a value of the overall sound pressure level that is somewhere near the mean. The accuracy of the estimated levels is expected to be within ± 5 dB, i.e. between approximately 56 per cent and 178 per cent of the actual r.m.s. pressure. Under reheat conditions or at jet velocities well above the datum velocity, however, the discrepancy could be larger.

The noise levels predicted by the method described here are free field values but do not apply to the field within a conical surface emanating from the jet nozzle perimeter and expanding at a semi-angle of approximately 15 degrees relative to the jet axis, since this region contains the turbulent mixing region of the jet.

This Section is also inapplicable to the case of "shock cell noise", a phenomenon which can occur when a jet nozzle is operating in an over-choked condition. A relatively discrete note is then produced, giving in a narrow band an intensity much greater than that of the basic jet noise.

2.3 Calculation Procedure

The following information is required as a basis for the computation of the sound pressure level at some point in the field:

jet nozzle diameter	D
co-ordinates of the point	(x,y)
mean fully expanded jet velocity	V
jet pipe total temperature	T_{Hj}.

(i) Evaluate x/D and y/D .

(ii) From Figure 1, which shows the free field datum noise level contours, read off the datum overall sound pressure level L_o at the position $(x/D, y/D)$.

(iii) From Figure 2, which shows velocity index contours, read off the velocity index n at the position $(x/D, y/D)$.

(iv) Calculate the first part of the velocity correction term ΔL_{v1} , from

$$\Delta L_{v1} = 10n \log_{10}\left(\frac{V}{V_o}\right) .$$

(v) Obtain the second part of the velocity correction term ΔL_{v2} from Figure 3, which shows ΔL_{v2} plotted against $\log_{10}(V/V_o)$ for different values of n .

(vi) Calculate the total jet velocity correction L_v from

$$L_v = \Delta L_{v1} + \Delta L_{v2} .$$

(vii) Calculate the density of the jet gases ρ_j ,

where $$\rho_j = \frac{p}{RT_j}$$

and $$T_j = T_{Hj} - \frac{V^2}{2c_p}$$ for all jet velocities.

(viii) Calculate density correction L_ρ ,

where $$L_\rho = 20 \log_{10}\left(\frac{\rho_j}{\rho_o}\right) .$$

(ix) Evaluate the overall sound pressure level (SPL) at the required point (x, y) as

$$L = L_o + L_v + L_\rho .$$

This level can be converted to a pressure loading by the use of the relation

$$\text{r.m.s. fluctuating pressure in N/m}^2 = 10^{(0.05L-4.699)}$$

or using Reference 2.5.1.

(x) For a given frequency the spectrum level in decibels relative to an arbitrary overall SPL may be read from Figure 4, after evaluating fD/V and x/D . Alternatively, if the spectrum level is required in terms of pressure loading, this may be found using Figure 5. Taking values of fD/V and x/D the spectrum level in $(N/m^2)/Hz$ relative to $1\ N/m^2$ overall SPL is read from the ordinate of Figure 5 and is then multiplied by the value of L obtained as a pressure loading in (ix). This gives the pressure spectrum level for a specified frequency in $(N/m^2)/Hz$.

2.4 Use of Calculated Sound Pressure Levels

After the free field sound pressure levels have been estimated, corrections must be made for local effects, for example reflection. When the wave fronts of this noise field strike a structure they are partially reflected, and the reflection process locally increases the pressure loading on the surface.

When the wave front strikes the surface at right angles (normal incidence) the pressure loading is doubled (a 6 dB increase on the calculated values). If the wave front moves parallel to the surface (grazing incidence) there is no increase in loading.

To allow for this reflection process, it is fairly general engineering practice to add a mean correction of 3 dB to the calculated free air levels.

The effects of different nozzle arrangements, aircraft configurations, etc., should be taken into consideration.

Further corrections must be made for the case of an aircraft in motion to allow for the effects of the aircraft velocity and ambient speed of sound.

An alternative computer-based method that plots overall sound pressure levels for various engine operating conditions can be seen in Derivation 2.5.3.

2.5 Derivation and Reference

Derivation

2.5.1	Franken, P.A. et al.	Methods of flight vehicle noise prediction. WADC TR 58-343, 1958.
2.5.2		Unpublished work by Rolls-Royce Ltd and Bristol Siddeley Engines Ltd.
2.5.3	Plumblee, H.E. et al.	Near field noise analyses of aircraft propulsion systems with emphasis on prediction techniques for jets. AFFDL-TR-67-43, 1967.
2.5.4	Riley, M.P.	Near field jet noise prediction techniques. British Aircraft Corporation Ltd, Acoustics Laboratory Report A.R.324, 1971.

Reference

2.5.5		The relation between sound pressure level and r.m.s. fluctuating pressure. Engineering Sciences Data Item No. 66018, 1966.

2.6 Example

It is required to estimate the r.m.s. sound pressure level and the pressure spectrum level at 300 Hz at a specific point given the following conditions:

$$D = 0.61 \text{ m}, \quad x = 5.5 \text{ m}, \quad y = 6.7 \text{ m},$$

$$V = 670 \text{ m/s}, \qquad T_{Hj} = 900 \text{ K},$$

$$V_o = 610 \text{ m/s}, \qquad \rho_o = 0.49 \text{ kg/m}^3,$$

$$R = 287 \text{ J/kg K}, \qquad c_p = 1160 \text{ J/kg K},$$

$$p = 101 \times 10^3 \text{ N/m}^2.$$

Firstly
$$\frac{x}{D} = \frac{5.5}{0.61} = 9.02 ,$$

and
$$\frac{y}{D} = \frac{6.7}{0.61} = 11.0 .$$

Hence, from Figure 1,
$$L_o = 140 \text{ dB}$$

and, from Figure 2, by interpolation
$$n = 6.2 .$$

Now
$$\Delta L_{v1} = 10 \times 6.2 \log_{10}\left(\frac{670}{610}\right) = 2.5 \text{ dB}$$

and, from Figure 3, for $\log_{10}(V/V_o) = 0.0407$ and interpolating for $n = 6.2$,
$$\Delta L_{v2} = -0.9 \text{ dB}$$

so that
$$L_v = 2.5 - 0.9 = 1.6 \text{ dB} .$$

As
$$T_j = 900 - \frac{670^2}{2 \times 1160} = 707 \text{ K}$$

then $$\rho_j = \frac{101 \times 10^3}{287 \times 707} = 0.498 \ \mathrm{kg/m^3} .$$

Now $$L_\rho = 20 \log_{10} \left(\frac{0.498}{0.49}\right) = 0.14 \ \mathrm{dB}$$

and so, finally, $$L = 140 + 1.6 + 0.14 = 142 \ \mathrm{dB} .$$

To determine the corresponding pressure spectrum level, for a frequency of 300 Hz, Figure 5 is used.

$$\frac{fD}{V} = \frac{300 \times 0.61}{670} = 0.273$$

and $$\frac{x}{D} = 9.02 .$$

By interpolation, the spectrum level relative to an overall sound pressure level of 1 N/m^2 is found to be 0.026 $(N/m^2)/Hz$.

In this Example $L = 142$ dB , which from Paragraph 3(ix) is equivalent to 252 N/m^2.

So the pressure spectrum level = 0.026 x 252 = 6.55 $(N/m^2)/Hz$.

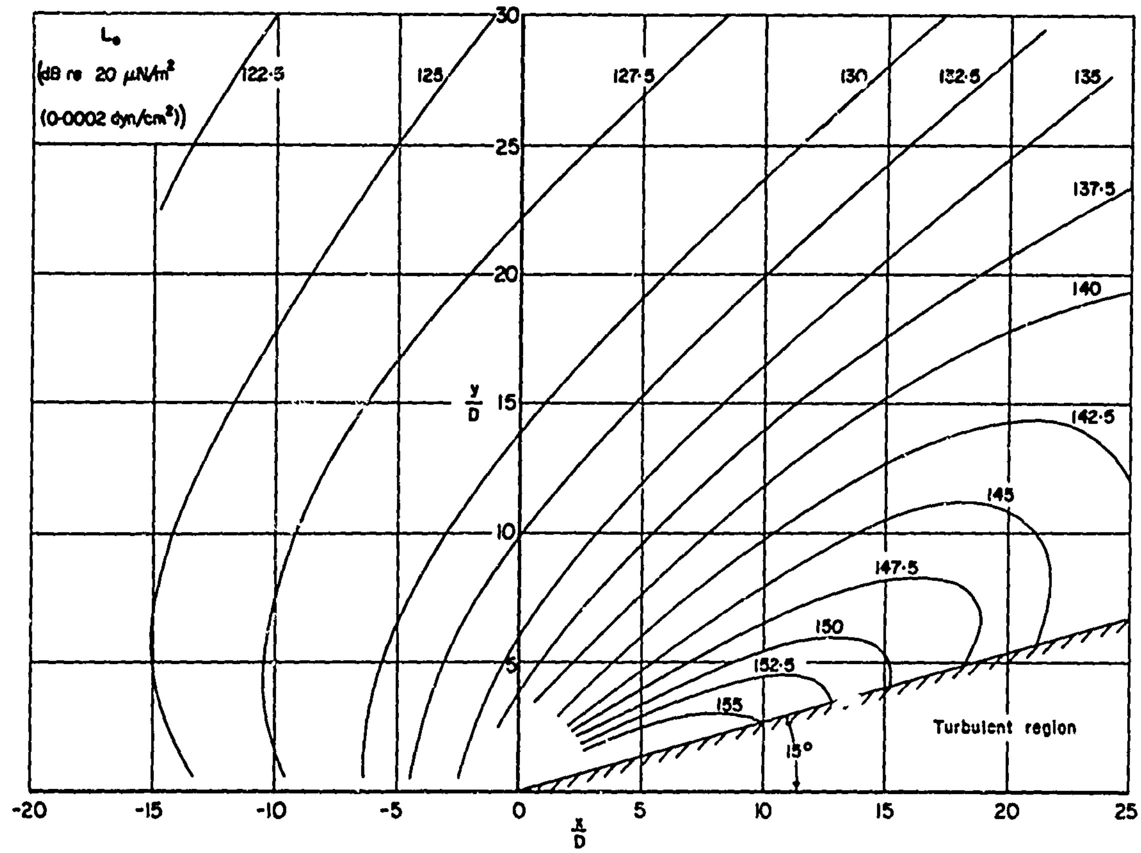


FIGURE 2.1

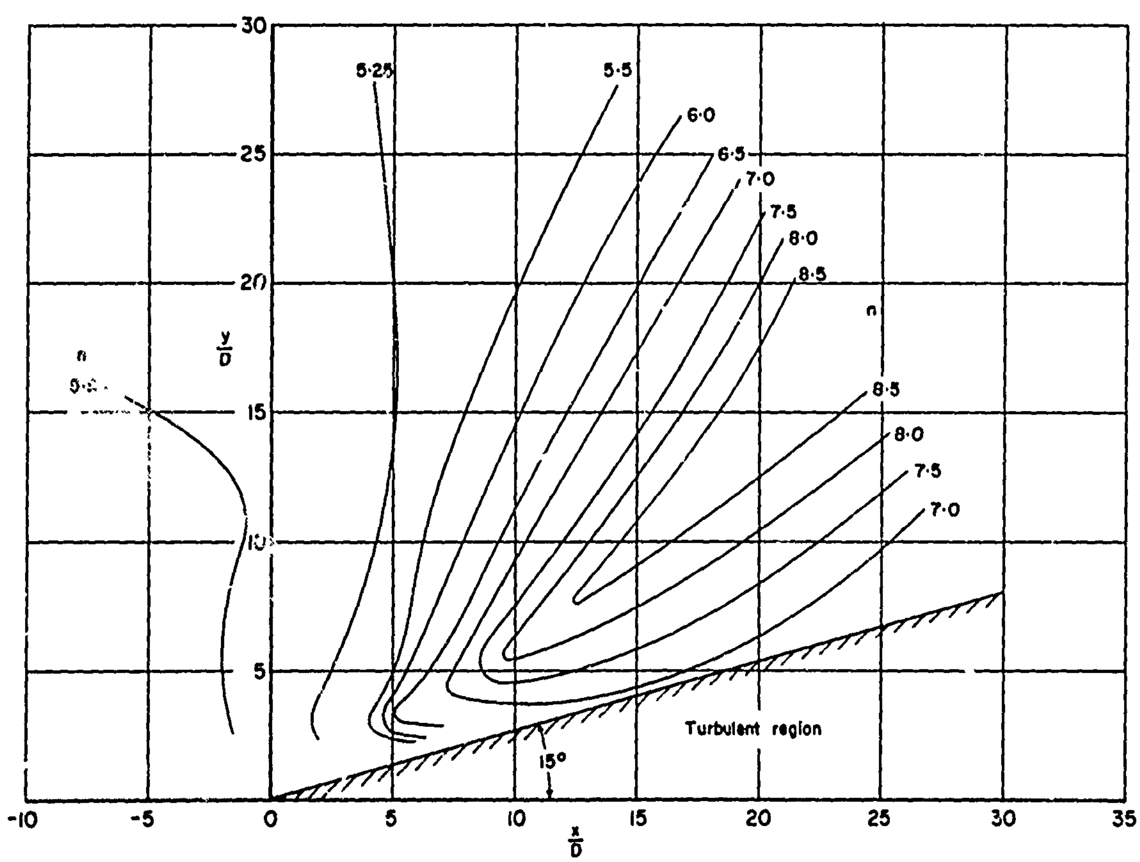
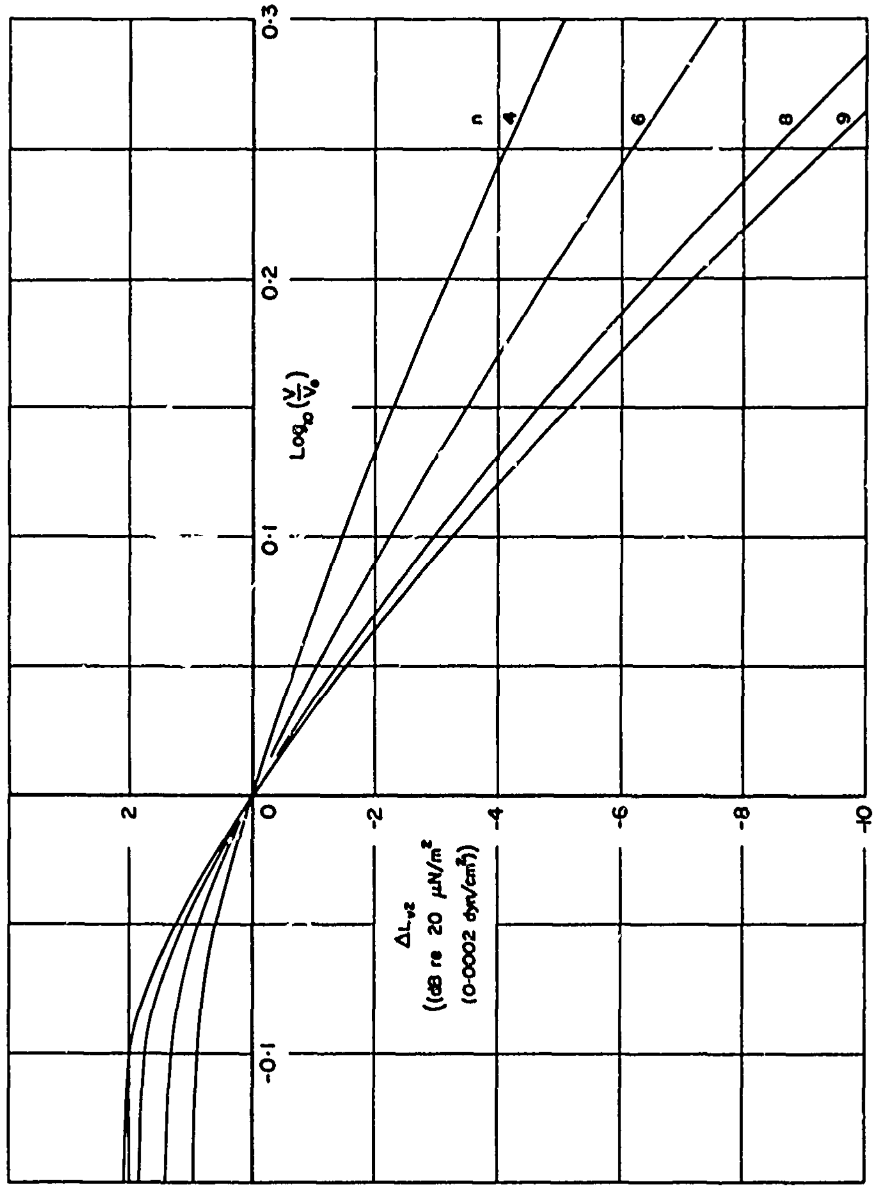


FIGURE 2.2



$Log_{10}(\frac{V}{V_0})$
-0·1
0·1
0·2
0·3
n
4
6
8
9
2
0
-2
-4
-6
-8
-10
ΔL_{vz}
(dB re 20 $\mu N/m^2$
(0·0002 dyn/cm²))

FIGURE 23

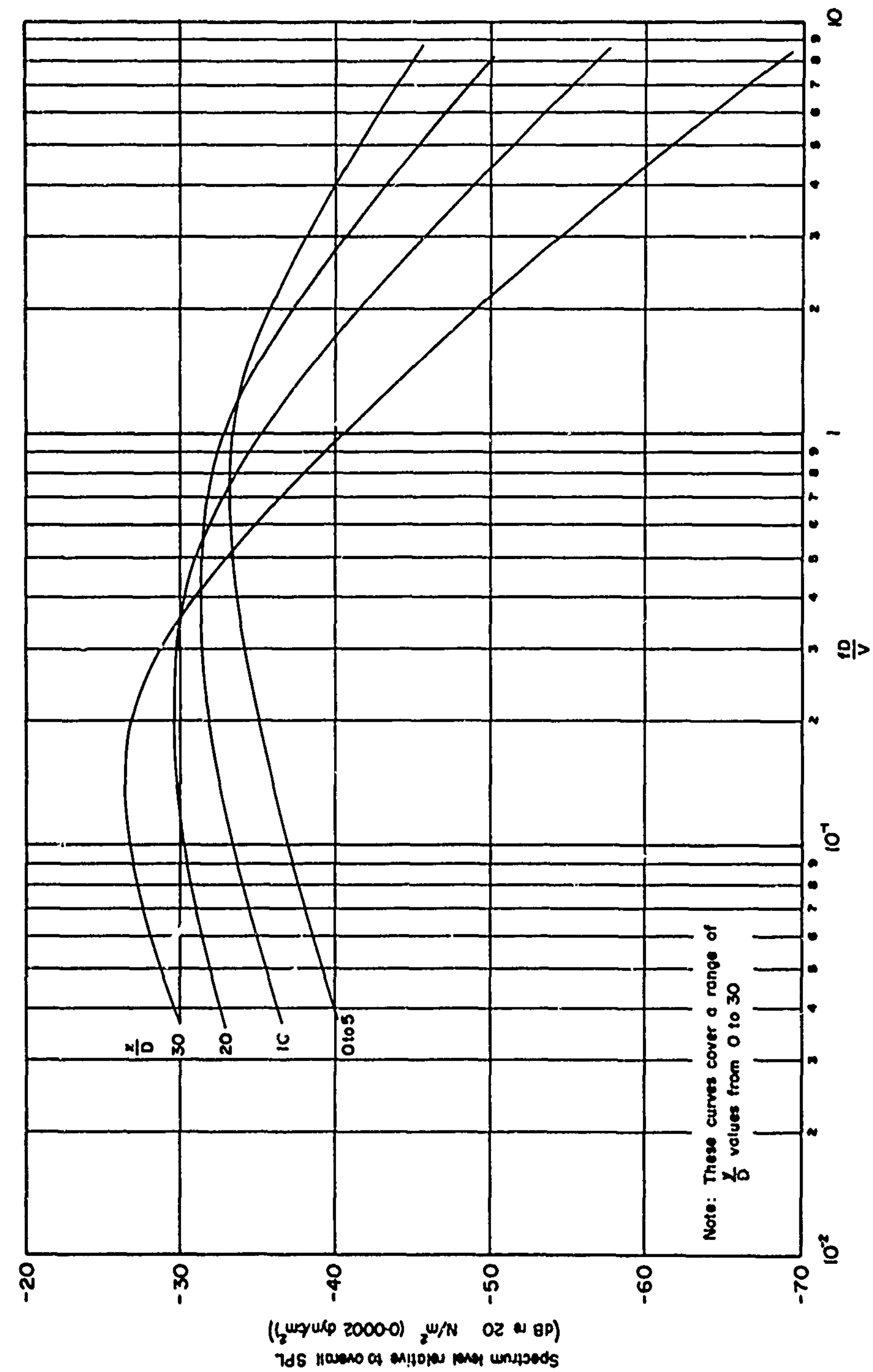
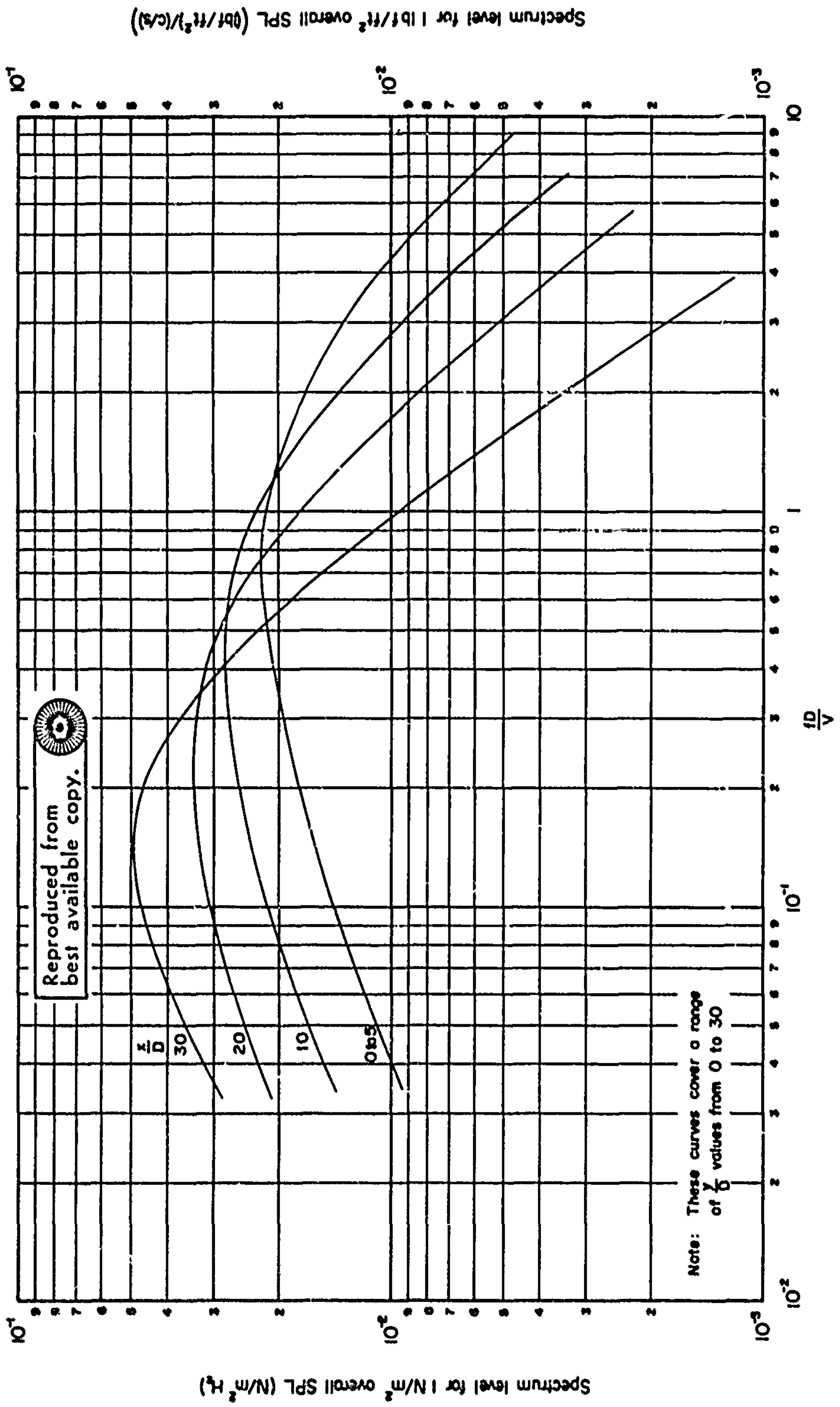

-20
-30
-40
-50
-60
-70
Spectrum level relative to overall SPL
(dB re 20 N/m² (0·0002 dyn/cm²))
10⁻²
10⁻¹
1
10
fD/V
x/D
30
20
10
0 to 5
Note: These curves cover a range of x/D values from 0 to 30

FIGURE 2.4



Spectrum level for 1 lbf/ft² overall SPL ((lbf/ft²)/(c/s))
Spectrum level for 1 N/m² overall SPL (N/m² Hz)
fD/V
x/D
30
20
10
0 to 5
Note: These curves cover a range of y/D values from 0 to 30
Reproduced from best available copy.

FIGURE 2.5

Section 3

NATURAL FREQUENCIES OF BUILT-UP, FLAT, PERIODIC SKIN STRINGER STRUCTURES

(Part I: Stringers rigid in bending)

3.1 Notation

a	width of skin plate or frame pitch	m	in
b	stringer pitch	m	in
D	flexural rigidity per unit width of the skin plate, $Et^3/\left[12(1-\sigma^2)\right]$	N m	lbf in
E	Young's modulus of skin material	N/m^2	lbf/in^2
E_s	Young's modulus of stringer material	N/m^2	lbf/in^2
f	frequency	Hz	c/s
G_s	shear modulus of stringer material	N/m^2	lbf/in^2
I_s	polar moment of inertia of stringer cross section about point on skin directly beneath shear centre of stringer	m^4	in^4
J_s	St Venant constant of uniform torsion for stringer cross section	m^4	in^4
K	frequency parameter	m/s	in/s
M_1	mode parameter for first group of natural frequencies		
M_2	mode parameter for second group of natural frequencies		
m	number of half waves across frame pitch		
N	number of spans		
r	mode number		
t	skin thickness	m	in
V	velocity parameter for skin material$^+$		
Γ_s	warping constant of stringer cross section with respect to the point on skin directly beneath shear centre of stringer	m^6	in^6
κ_R	non-dimensional torsional stiffness of stringer = actual rotational stiffness per unit length of the stringer $\times b/D$		
ρ	density of skin material	kg/m^3	*
ρ_s	density of stringer material	kg/m^3	*
σ	Poisson's ratio of the skin material		

Both SI and British units are quoted but any coherent system of units may be used.

$^+$ The velocity parameter is defined in SI units as $V = (E/\rho)^{1/2}/5080$ and in British units as $V = (E/\rho)^{1/2}/200\ 000$. V is approximately unity for all common structural metallic materials.

* A density value expressed in British units as pounds per cubic inch has to be divided by 386.4 before it can be used in the formula given here. (A force of 1 lbf acting on a mass of 1 lb produces an acceleration of 386.4 in/s^2.)

3.2 Introduction

This Section gives the natural frequencies of vibration of built-up, flat skin-stringer structures with equispaced and identical stringers as shown in Figures 3.1 and 3.2. The natural frequencies of such multi-span structures occur in groups, the number of frequencies in each group being equal to the number of spans. For a panel having a large number of spans the lowest natural frequency in any group is generally associated with a mode in which the stringers twist and the highest natural frequency in that group is associated with a mode in which the stringers tend to bend without twisting. For a panel having a small number of spans, typical mode shapes, as shown in Figure 3.3, are affected by the edge conditions. The mode shapes alternate between symmetric and anti-symmetric as frequency increases.

In the Derivation the frame torsional stiffness has been neglected and the frame bending stiffness has been assumed to be infinite. The skin edges at the frames are thus assumed to be simply-supported. The natural frequencies are only slightly influenced by the stringer bending stiffness and therefore this stiffness has been assumed to be infinity.

This Section can be used to predict the first two groups of natural frequencies of skin-stringer structures with four different end conditions. The results have been presented for varying degrees of stringer torsional stiffness, for three different aspect ratios (a/b) and for two different values of m, the number of half-waves across the frame pitch.

3.3 Calculation Procedure

Calculate f from

$$f = VK \frac{t}{b^2} .$$

<u>To determine K</u>

(i) Calculate κ_R from the equation

$$\kappa_R = \left\{ E_s \Gamma_s \left(\frac{m\pi}{a}\right)^4 + G_s J_s \left(\frac{m\pi}{a}\right)^2 - \rho_s I_s (2\pi f)^2 \right\} \times \frac{b}{D} .$$

Since f is unknown its value has to be estimated initially. Values of $f = 100$ Hz for the first group of frequencies and $f = 200$ Hz for the second group should give results of sufficient accuracy for commonly used skin-stringer structures. Alternatively, for the first group of natural frequencies, f may be taken as the fundamental natural frequency of any individual panel with fully-fixed edges. For the second group, f may be taken as the natural frequency of the same panel with fully-fixed edges in its second mode (i.e. with approximately two half waves along the length b). Both these frequencies can be calculated using Reference 3.5.3.

(ii) Calculate M_1 or M_2 from the following table:

End Conditions as in	M_1 $(1 \leqslant r \leqslant N)$	M_2 $(N+1 \leqslant r \leqslant 2N)$
Figure 2a	$\frac{r-1}{N}$	$2 - \frac{r-1}{N}$
Figure 2b	$\frac{r}{N}$	$2 - \frac{r}{N}$
Figure 2c	$\frac{2r-1}{2N}$	$2 - \frac{2r-1}{2N}$
Figure 2d	see Note in Step (iii) below	

(iii) Read K from Figures 4-9 for appropriate values of m, aspect ratio (a/b), κ_R and M_1 or M_2.

Note: For structures with end conditions as in Figure 2d, K is approximately equal to the arithmetic average of the values of K obtained from the first two rows in the above table.

3.4 Notes

The effect of including the frame torsional stiffness would be to increase the natural frequencies slightly and this would be more pronounced for structures with smaller aspect ratios. The effect of including the frame bending stiffness would be to decrease the natural frequencies slightly.

For structures with a low stringer torsional stiffness, the bounding frequencies of the first group (with $m = 1$ or 2) can also be found from Data Item No. 66019.

Derivations 3.5.2, 3.5.4 and 3.5.5 give the transfer-matrix method of analysis of skin-stringer structures which could be used to calculate the natural frequencies of non-periodic structures, i.e. structures with non-uniform skin/stringer characteristics.

3.5 Derivation

3.5.1	Lin, Y.K. Brown, I.D. Deutschle, P.C.	Free vibration of a finite row of continuous skin-stringer panels. J. Sound and Vibration, Vol.1, No.1, January 1964.
3.5.2	Lin, Y.K. et al.	Free vibration of continuous skin-stringer panels with non-uniform stringer spacing and panel thickness. AFML-TR-64-347, February 1965.
3.5.3		Natural frequencies of uniform flat plates. Engineering Sciences Data Item No. 66019, 1966.
3.5.4	Mercer, C.A. Seavey, C.	Prediction of natural frequencies and normal modes of skin-stringer panel rows. J. Sound and Vibration, Vol.6, No.1, January 1967.
3.5.5	Mercer, C.A. Seavey, C.	Program for the calculation of natural frequencies and normal modes of skin-stringer panel arrays. Institute of Sound and Vibration Research Tech. Report No.6, July 1968.
3.5.6	Sen Gupta, G.	Natural flexural waves and the normal modes of periodically supported beams and plates. J. Sound and Vibration, Vol.13, No.1, September 1970.
3.5.7	Sen Gupta, G.	Natural frequencies of periodic skin stringer structures using a wave approach. J. Sound and Vibration, Vol.16, No.4, June 1971.

3.6 Example

It is required to determine the first group of natural frequencies of a six-span skin-stringer structure having the following dimensions etc. with end conditions as in Figure 2d and with a single half wave across the skin width.

$a = 508$ mm	$\Gamma_s = 3.1 \times 10^{-12}\ \mathrm{m}^6$
$b = 254$ mm	$I_s = 10.5 \times 10^{-8}\ \mathrm{m}^4$
$t = 1$ mm	$E = E_s = 72\ 300\ \mathrm{MN/m}^2$
$\sigma = 0.3$	$\rho = \rho_s = 2800\ \mathrm{kg/m}^3$
$J_s = 94.4 \times 10^{-12}\ \mathrm{m}^4$	$G_s = 27\ 800\ \mathrm{MN/m}^2$.

From this $D = \dfrac{Et^3}{12(1-\sigma^2)} = 6.62$ N m, $\quad V = \dfrac{(E/\rho)^{1/2}}{5080} = 1.00$

and for the first group of natural frequencies with a single half wave across the skin width ($m = 1$ and assuming $f = 100$ Hz),

$$\kappa_R = \left\{E_s\Gamma_s\left(\frac{\pi}{a}\right)^4 + G_sJ_s\left(\frac{\pi}{a}\right)^2 - \rho_sI_s(2\pi \times 100)^2\right\} \times \frac{b}{D}$$

$$= (328 + 100 - 116) \times 0.0384$$

$$= 12.0.$$

For this group of frequencies, with $m = 1$, $a/b = 2.0$, $\kappa_R = 12.0$ and using Figure 5a, the results can be calculated and presented in the form of the following

Mode number	End conditions as in Figure 2a		End conditions as in Figure 2b		End conditions as in Figure 2d	$f = VK \frac{t}{b^2}$
r	M_1	K (from Figure 5a) (m/s)	M_1	K (from Figure 5a) (m/s)	K (Average of the results in columns III and V) (m/s)	(using K from column VI) (Hz)
(I)	(II)	(III)	(IV)	(V)	(VI)	(VII)
1	0	4270	1/6	4360	4315	66.9
2	1/6	4360	1/3	4610	4485	69.5
3	1/3	4610	1/2	4970	4790	74.2
4	1/2	4970	2/3	5370	5170	80.1
5	2/3	5370	5/6	5690	5530	85.7
6	5/6	5690	1	5930	5760	89.3

Similar calculations can be done for a structure with end conditions as in Figure 2c. It may be noted that the frequencies obtained in column VII are less than 100 Hz and therefore the value of κ_R has been slightly under-estimated. This means the actual frequencies are slightly higher in this particular problem. Further calculations show that the calculated frequencies are within about 3.5 per cent of the frequencies obtained by taking a further iteration.

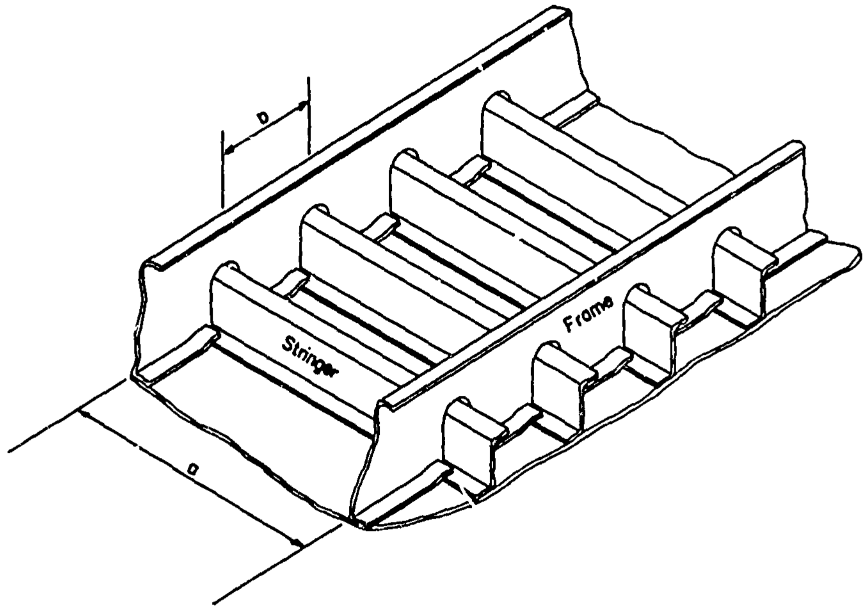


FIGURE 31. A PERIODIC SKIN STRINGER STRUCTURE

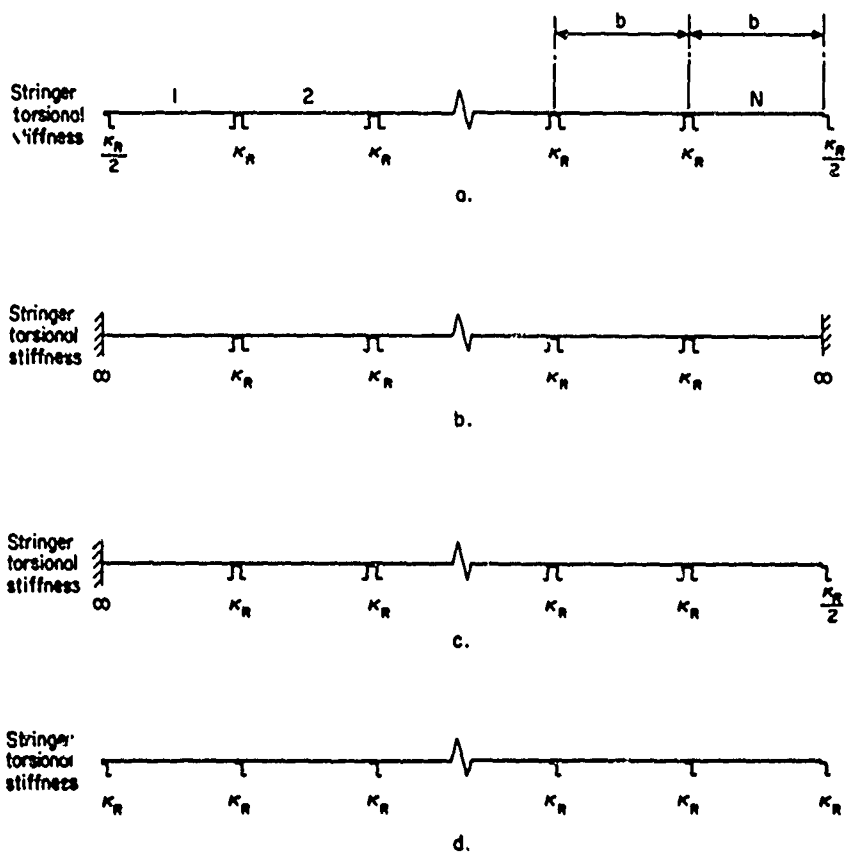


FIGURE 32. FINITE SKIN-STRINGER STRUCTURES WITH DIFFERENT BOUNDARY CONDITIONS

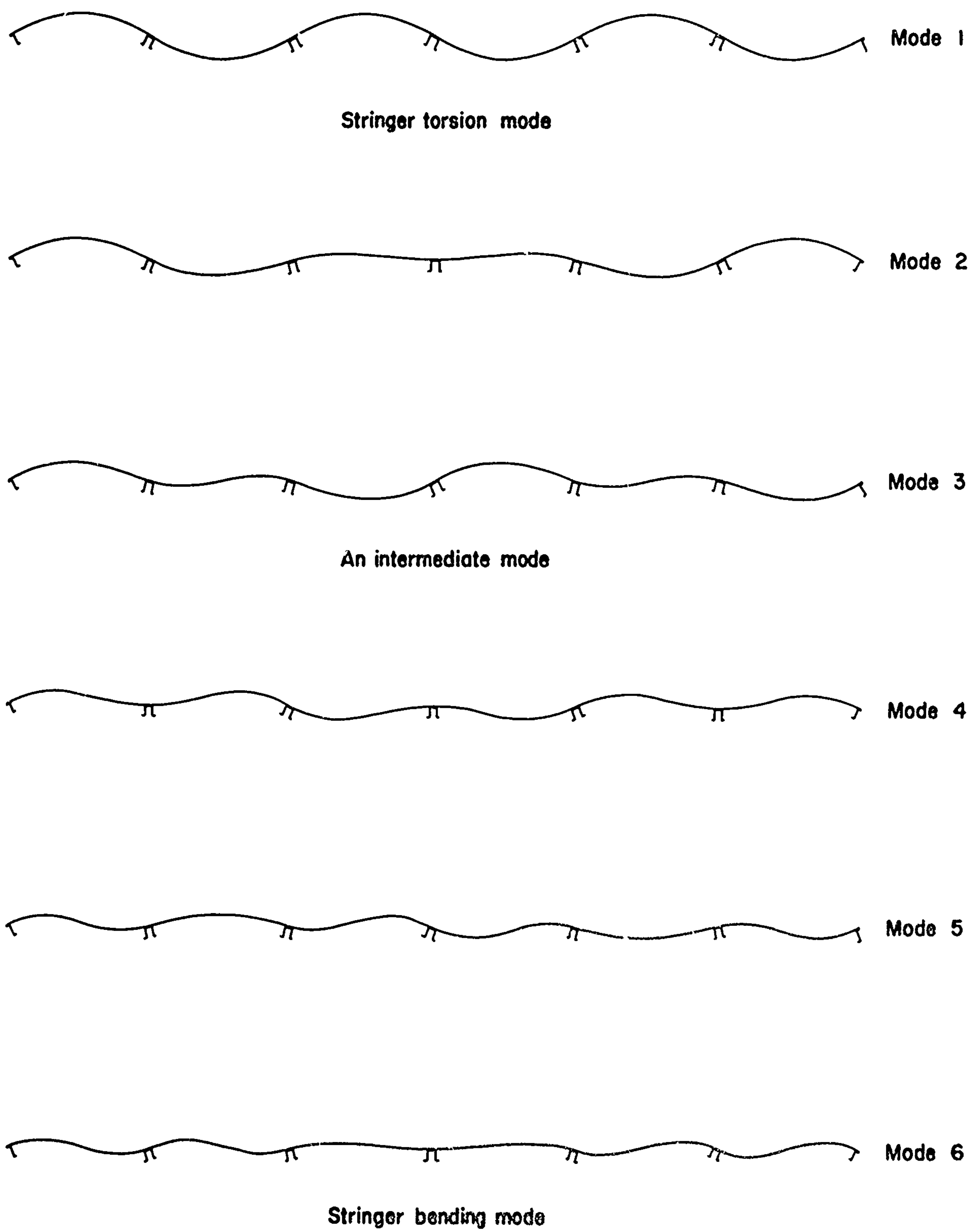


FIGURE 33. TYPICAL MODE SHAPES OF A SIX-SPAN SKIN-STRINGER PANEL
(ONLY THE FIRST GROUP IS SHOWN)

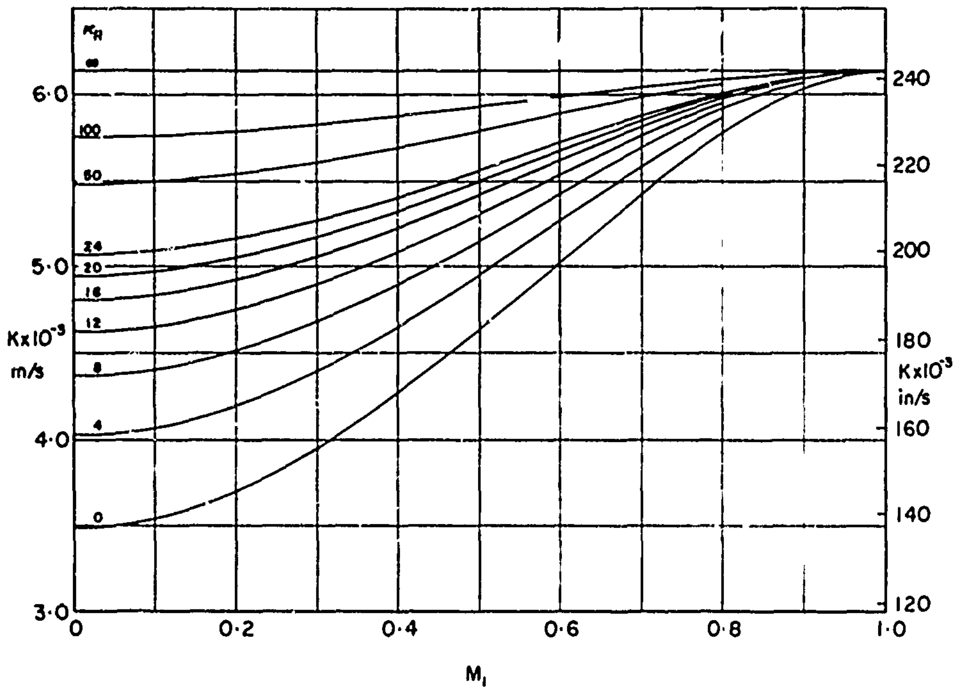


FIGURE 3.4. $\frac{a}{b} = 1{\cdot}5$, m=1, GROUP I

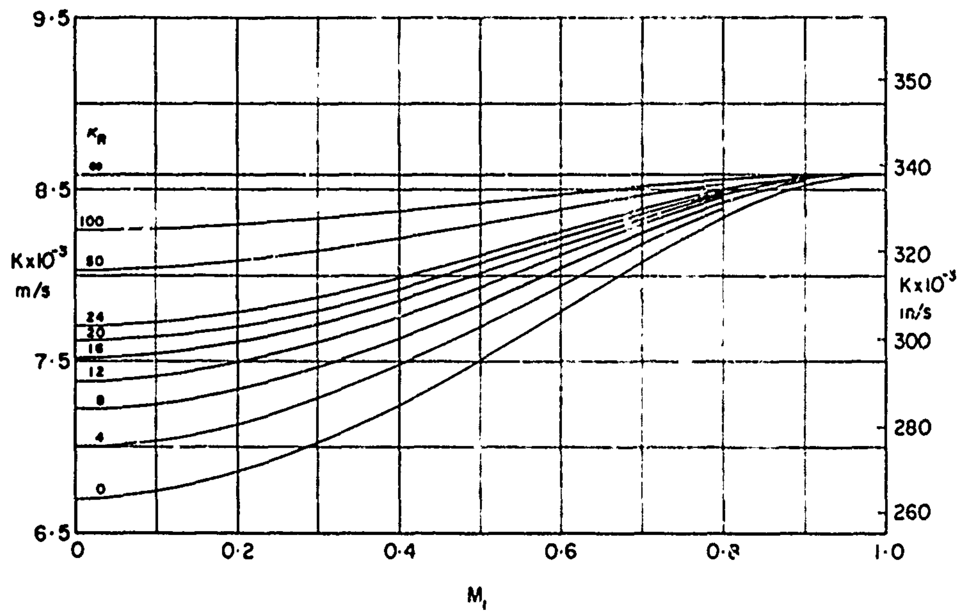


FIGURE 3.5. $\frac{a}{b} = 1{\cdot}5$, m=2, GROUP I

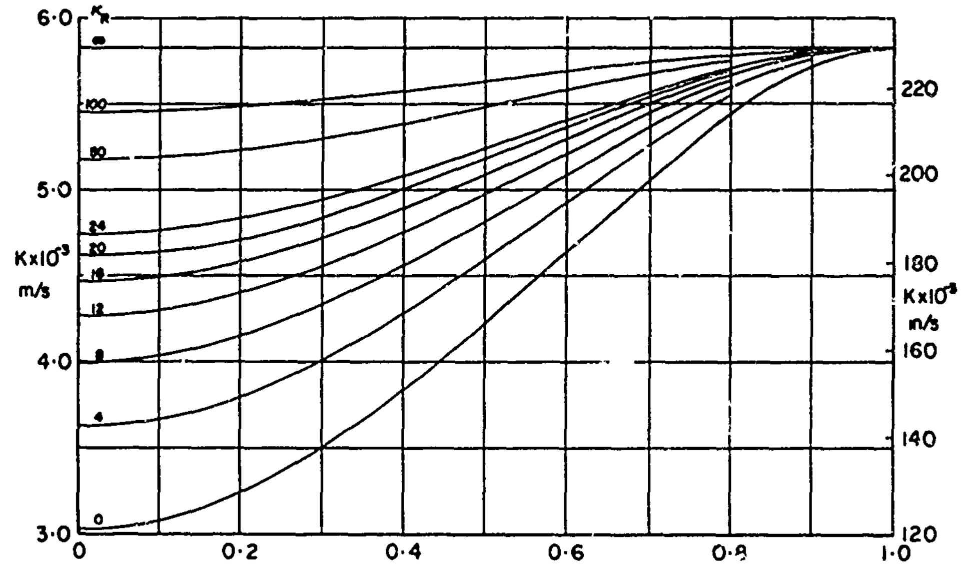


FIGURE 3.6. $\frac{a}{b}$ = 2·0, m = 1, GROUP I

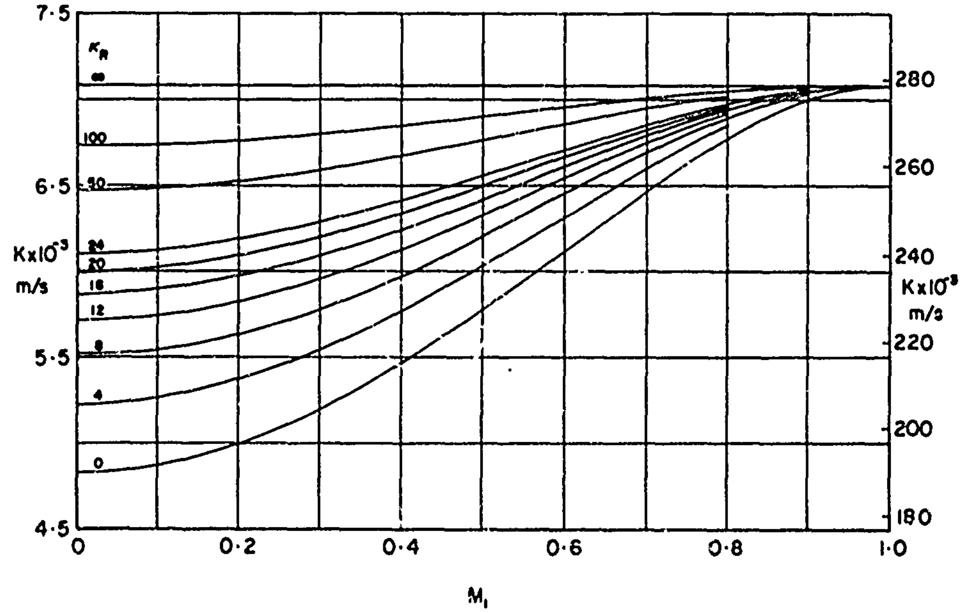


FIGURE 3.7. $\frac{a}{b}$ = 2·0, m = 2, GROUP I

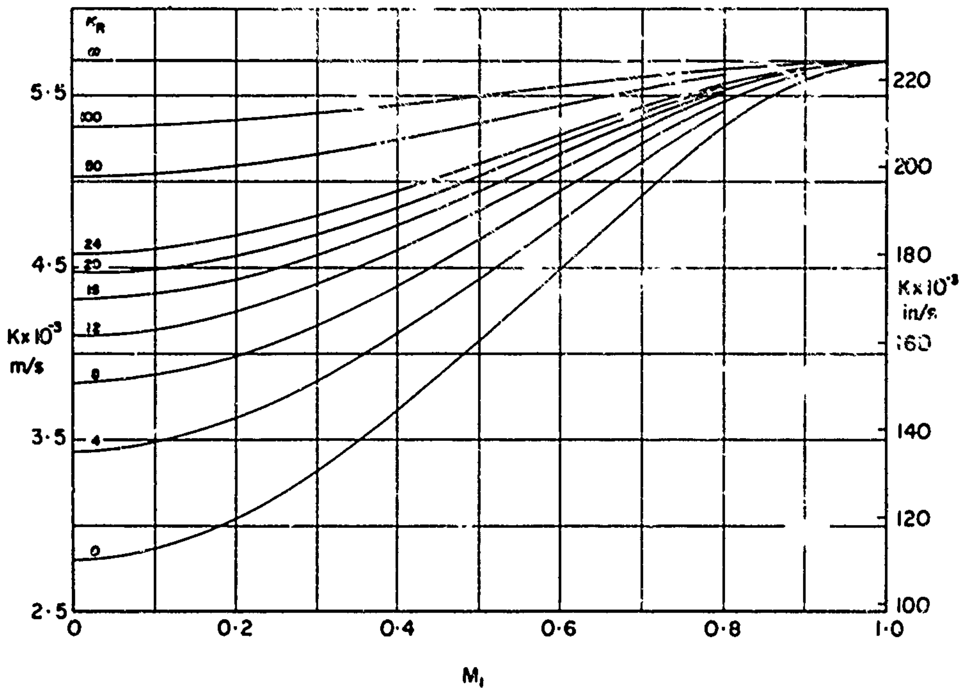


FIGURE 3.8. $\frac{a}{b} = 2{\cdot}5$, m=1, GROUP I

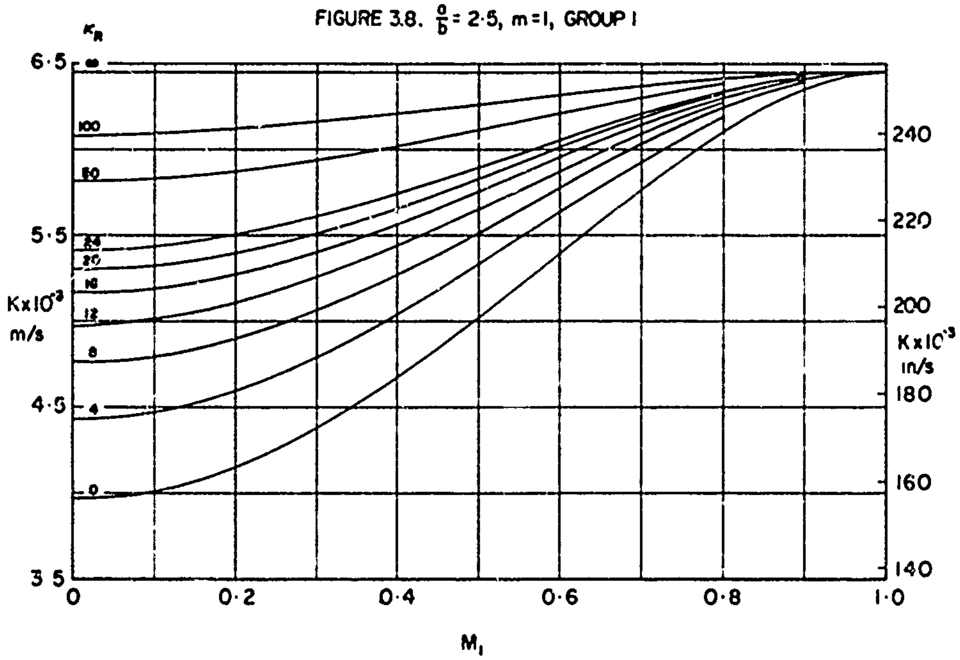


FIGURE 3.9. $\frac{a}{b} = 2{\cdot}5$, m=1, GROUP I

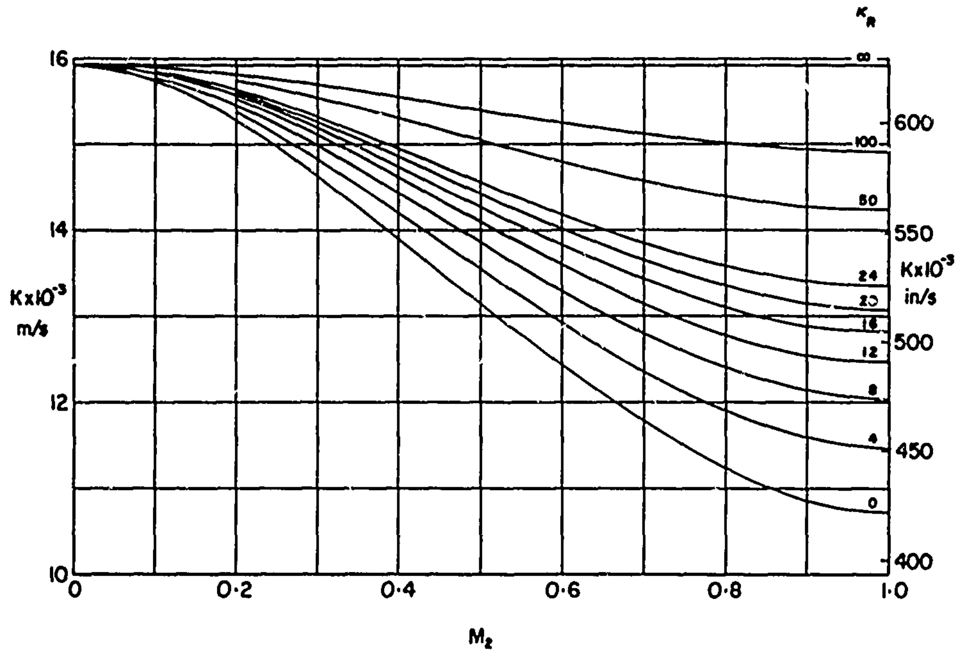


FIGURE 3.10. $\frac{a}{b}=1{\cdot}5$, m=1, GROUP 2

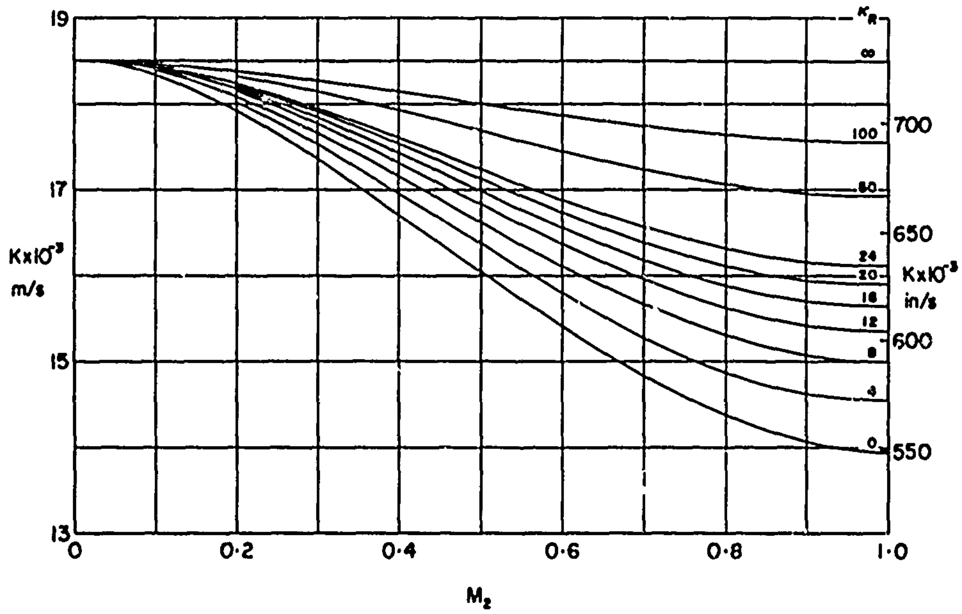


FIGURE 3.11. $\frac{a}{b}=1{\cdot}5$, m=2, GROUP 2

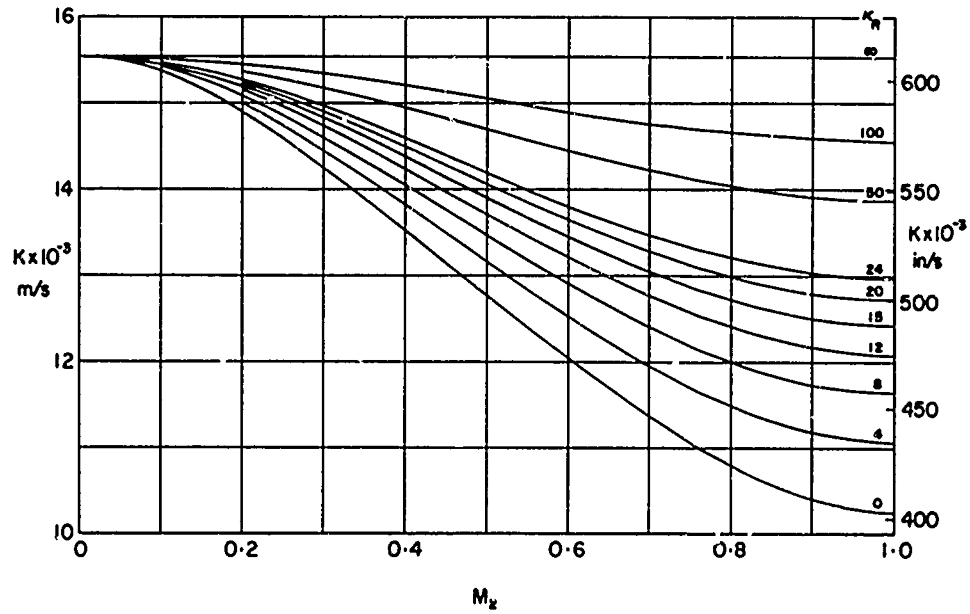


FIGURE 3.12. $\frac{a}{b} = 2{\cdot}0$, m=1, GROUP 2

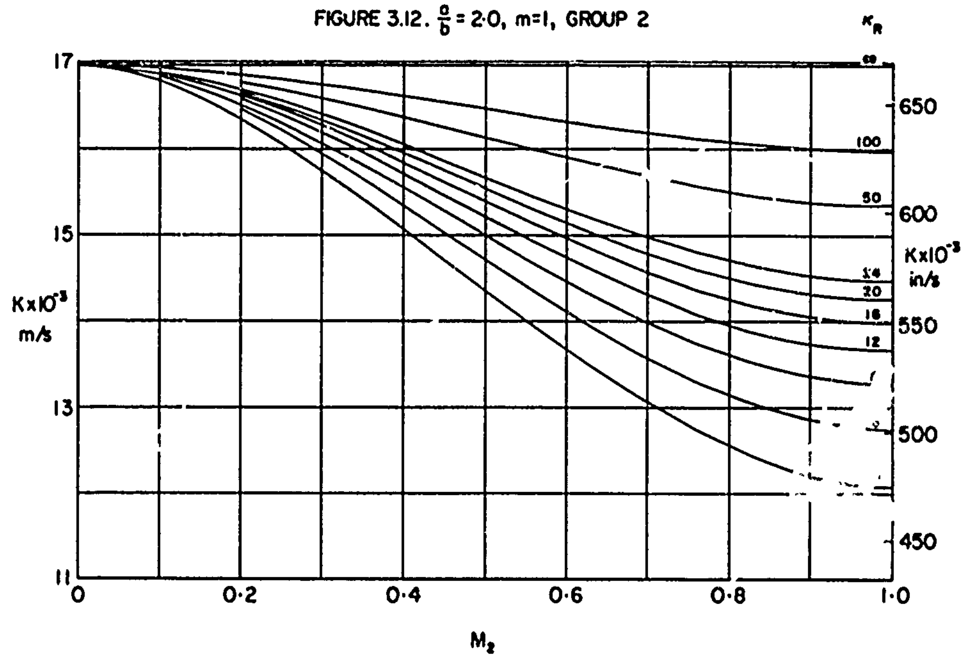


FIGURE 3.13. $\frac{a}{b} = 2{\cdot}0$, m=2, GROUP 2

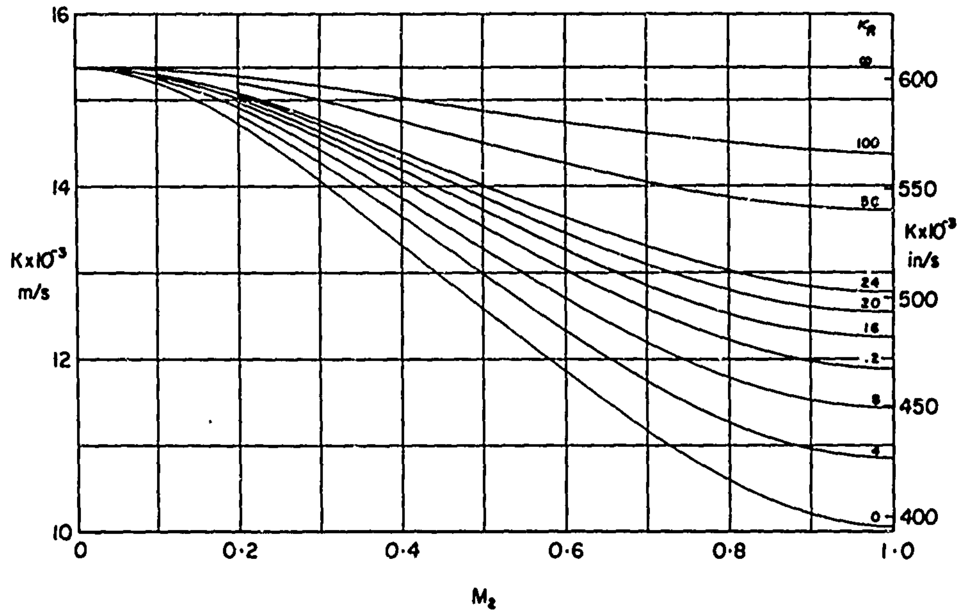


FIGURE 3.14. $\frac{a}{b} = 2{\cdot}5$, m=1, GROUP 2

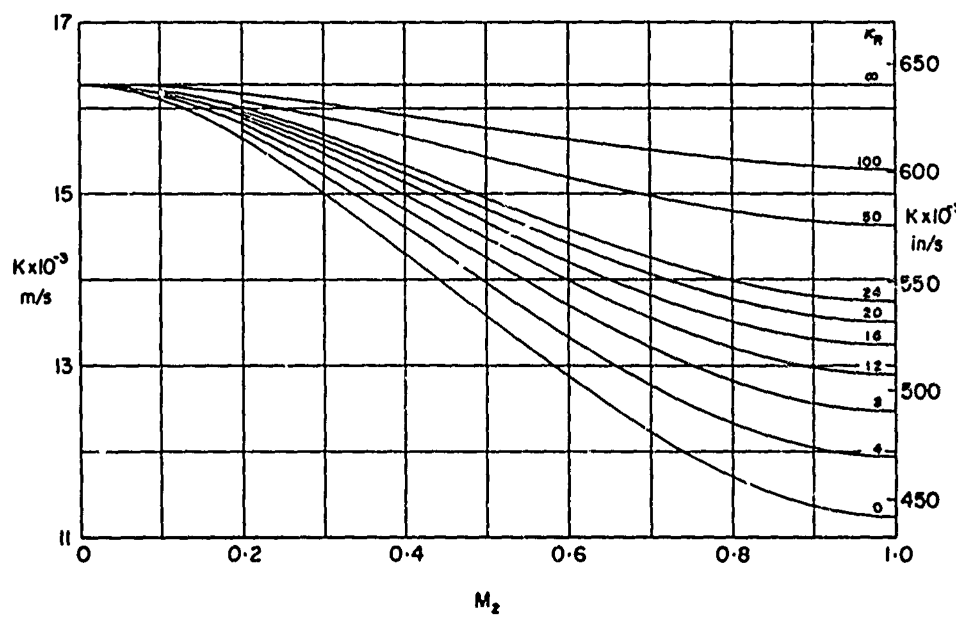


FIGURE 3.15 $\frac{a}{b} = 2{\cdot}5$, m=2, GROUP 2

Section 4

NATURAL FREQUENCIES OF RECTANGULAR SINGLY-CURVED PLATES

4.1 Notation

a	length of longer plate side	m	in
b	arc length of shorter plate side	m	in
E	Young's modulus of plate material	N/m^2	lbf/in^2
f	natural frequency of vibration of plate	Hz	c/s
K	natural frequency parameter for fixed edge plates	m/s	in/s
$K_{m,n}$	natural frequency parameter of simply-supported plate in (m,n)th mode	m/s	in/s
m	number of half waves in plate parallel to longer side		
n	number of half waves in plate parallel to shorter (curved) side		
R	radius of curvature of plate	m	in
t	thickness of plate	m	in
V	velocity parameter for plate material+		
σ	Poisson's ratio of plate material		
ρ	density of plate material	kg/m^3	*

Both SI and British units are quoted but any coherent system of units may be used.

4.2 Notes

4.2.1 Plates with simply-supported edges

The natural frequency is given by

$$f = V K_{m,n} \frac{n^2 t}{b^2} .$$

In Figure 4.1 values of $K_{m,n}$ are plotted against an/mb for a range of values of b^2/n^2Rt for initially unstressed, cylindrically curved, rectangular plates of uniform thickness having all edges simply supported. For plates having all edges simply supported the modes are exactly defined by the number of half waves across the length and breadth of the panel. These natural frequencies can be presented on one figure since the condition at a nodal line is the same as that at the boundaries parallel to the nodal line.

4.2.2 Plates with fixed edges

The natural frequency is given by

$$f = V K \frac{t}{b^2} .$$

+ The velocity parameter is defined in SI units by $V = (E/\rho)^{\frac{1}{2}}/5080$ or in British units by $V = (E/\rho)^{\frac{1}{2}}/200\,000$. V is approximately unity for all common structural metallic materials.

* A density value expressed in British units as pounds per cubic inch has to be divided by 386.4 before it can be used in the formula for V given here. (A force of 1 lbf acting on a mass of 1 lb produces an acceleration of 386.4 in/s^2.)

In Figures 4.2 to 4.8 values of K are plotted against a/b for a range of values of b^2/Rt for a number of natural modes for initially unstressed, cylindrically curved, rectangular plates of uniform thickness having all edges fully fixed. These modes of vibration are not necessarily uniquely identified by the number of half waves across the length and breadth of the plate. In this Section the fixed-edge plate modes are defined by the symmetry or antisymmetry in two directions; stating first the symmetry condition parallel to the plate longer side followed by the symmetry condition parallel to the plate shorter curved side. The order of modes in any mode type is given in ascending order of frequency.

The fixed edge panel modes condidered in this Section are listed in Table I.

Table I

Mode	Figure for frequency parameter	Figure for mode shape
First (S,S)	4.2	4.9
Second (S,S)	4.3	4.10
First (S,A)	4.4	4.11
Second (S,A)	4.5	4.12
First (A,S)	4.6	4.13
Second (A,S)	4.7	4.14
First (A,A)	4.8	4.15

S = symmetric A = antisymmetric

The expression used to calculate the fixed-edge plate frequencies contains parameters a/b, ba/Rt and b/R. The curves in this section are for a value of b/R of 0.1, but the curves may be used for shallow plates generally. At a value of b/R of 1.0 the frequencies are 2-3 per cent less than obtained from this Section.

4.2.3 General notes

In deriving the curves the value of σ was assumed to be 0.3. This value is sufficiently accurate for all common structural metallic materials.

In the theory on which this Section is based it is assumed that panels are thin and of shallow curvature. These conditions are satisfied when $R/t > 20$ and $b/R < 1.5$.

Shear deflection and rotary inertia have been neglected when calculating the curves for both simply-supported and fixed-edge plates. This is a generally accepted assumption if the ratio of half wave lengths to plate thickness is greater than 10.

For flat plates with simply-supported edges the lowest natural frequency of vibration occurs in the mode having a single half wave across the length and breadth of the plate, i.e. the (1,1) mode. For flat plates with fixed edges the lowest natural frequency occurs in the first symmetric - symmetric mode where the mode shape is similar to the simply-supported edge (1,1) mode. As the curvature of the plate increases (radius reduces) the lowest frequency, for both simply-supported and fixed-edge plates, tends to occur in a mode where the number of half waves on the curved side is greater than one.

4.3 Application to Aircraft Structural Panels

In typical aircraft construction, stiffening members such as ribs, frames and stringers divide the panel into plates. Owing to the mechanical coupling between plates, normal modes result in which arrays of plates vibrate together. Theoretical analysis indicates that an array of plates has groups of resonant frequencies. Each group is bounded by a lower frequency mode where stringers twist and a higher frequency mode where stringers bend. These lower and higher frequency modes correspond to the single plate modes with simply-supported and fixed edges respectively. In between these two bounding frequencies there is a number of intermediate modes in which the stringers both twist and bend. The second and subsequent frequency groups correspond to overtone modes of the fundamental group.

The number of plates showing predominant vibration at any one natural frequency of the group depends on the relative dimensions of the plates and the phase relationships of the exciting force over the surface of the panel.

The lowest natural frequency is generally associated with a mode of vibration in which the response of plates is such that half waves on the two sides of a stringer are out of phase. If the stringers are of low torsional stiffness, the frequency for the panel will be close to the value indicated by this Section for a single plate with simply-supported edges. With increasing torsional stiffness the lowest resonant frequency of the group rises to approach the value indicated by this Section for a single plate with fixed edges.

The highest natural frequency of a panel in a frequency group is associated with a mode of vibration in which the half waves in the plates on the two sides of a stringer are in phase. This behaviour corresponds to fixed-edge conditions.

4.4 Derivation

4.4.1	Timoshenko, S.	Vibration problems in engineering. Second edition. Von Nostrand, New York, 1937.
4.4.2	Warburton, G.B.	The vibration of rectangular plates. Proc. I.Mech.E., Vol.168, No.12, pp.371-384, 1954.
4.4.3	Clarkson, B.L.	The design of structures to resist jet noise fatigue. J. R. aeronaut. Soc., Vol.66, No.622, pp.603-616, October 1962.
4.4.4	Sewall, J.L.	Vibration analysis of cylindrically curved panels with simply supported or clamped edges and comparison with some experiments. NASA tech. Note D-3791, January 1967.
4.4.5	Webster, J.J.	Free vibrations of rectangular curved panels. Int. J. Mech. Sci., Vol.10, No.7, pp.571-582, July 1968.
4.4.6	Petyt, M.	Vibration of curved plates. J. Sound Vib., Vol.15, No.3, pp.381-395, April 1971.
4.4.7	Webster, J.J. Warburton, G.B.	Discussion of "Approximate methods for the determination of the natural frequencies of stiffened and curved plates". J. Sound Vib., Vol.18, No.1, pp.139-141, September 1971.

4.5 Example

It is required to estimate the lowest natural frequency of an aluminium alloy plate in both the simply-supported and fixed-edge conditions. The plate has the following dimensions and physical properties:

$a = 300$ mm, $b = 200$ mm, $R = 1000$ mm,

$t = 0.8$ mm, $E = 70\ 000$ MN/m^2, $\rho = 2770$ kg/m^3.

Firstly $$\frac{a}{b} = \frac{300}{200} = 1.5\ ,$$

$$\frac{b^2}{Rt} = \frac{200^2}{1000 \times 0.8} = 50$$

and $$V = \sqrt{\frac{70\ 000 \times 10^6}{2770}} \times \frac{1}{5080} = 0.99.$$

4.5.1 All edges simply-supported

Obtain the natural frequency parameters from Figure 4.1.

Mode (m,n)	$\frac{a}{m}\frac{n}{b}$	$\frac{b^2}{n^2Rt}$	$10^{-3}K_{m,n}$ (m/s)	$\frac{n^2t}{b^2}$ (m^{-1})	$f = VK_{m,n}\frac{n^2t}{b^2}$ (Hz)
1,1	1.5	50	12.9	0.02	255
1,2	3.0	12.5	2.87	0.08	227
2,1	0.75	50	26.7	0.02	529
1,3	4.5	5.556	2.54	0.18	453
3,1	0.5	50	34.5	0.02	683

Hence the lowest natural frequency with all edges simply supported occurs in the (1,2) mode and is 227 Hz.

4.5.2 All edges fixed

For lowest symmetric-symmetric mode from Figure 4.2

$$10^{-3}\,K \quad = \quad 27.3 \quad m/s\ .$$

For lowest symmetric-antisymmetric mode from Figure 4.4

$$10^{-3}\,K \quad = \quad 20.0 \quad m/s\ .$$

For lowest antisymmetric-symmetric mode from Figure 4.6

$$10^{-3}\,K \quad = \quad 31.2 \quad m/s\ .$$

For lowest antisymmetric-antisymmetric mode from Figure 4.8

$$10^{-3}\,K \quad = \quad 28.0 \quad m/s\ .$$

Hence the lowest natural frequency with all edges fixed occurs in the first symmetric-antisymmetric mode

and is $$0.99 \times 20.0 \times 10^3 \times \frac{0.8 \times 1000}{200^2} \quad = \quad 396 \quad Hz\ .$$

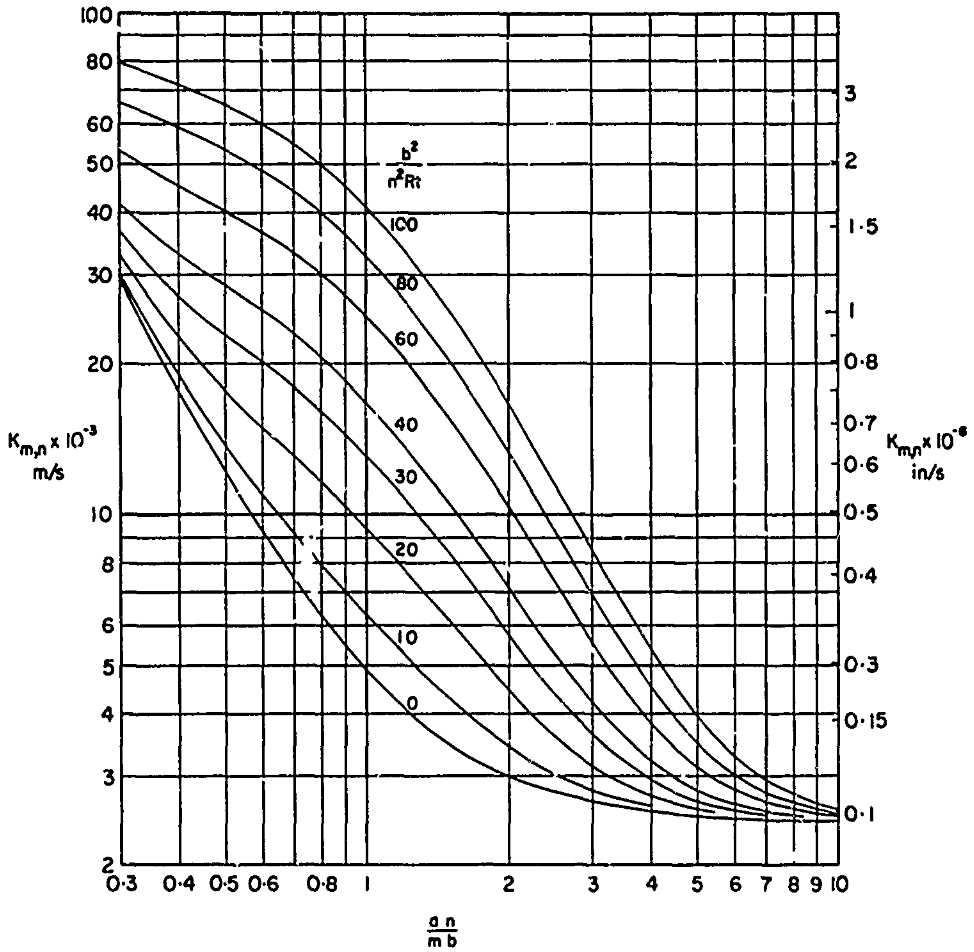


FIGURE 4.1. FREQUENCY PARAMETERS FOR PLATES WITH ALL EDGES SIMPLY-SUPPORTED

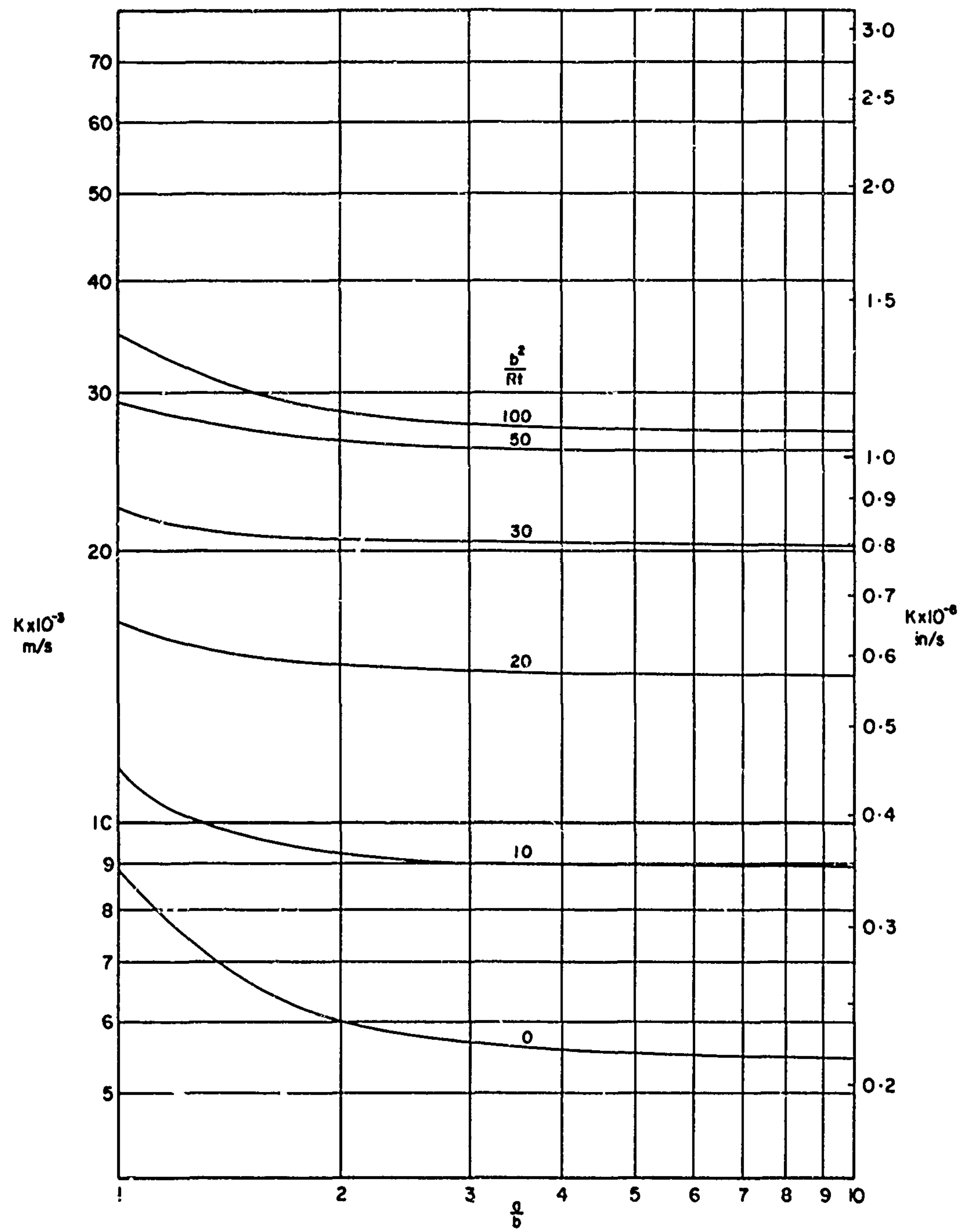


FIGURE 4.2. FREQUENCY PARAMETER FOR FIRST SYMMETRIC-SYMMETRIC MODE (PLATES WITH ALL EDGES FIXED)

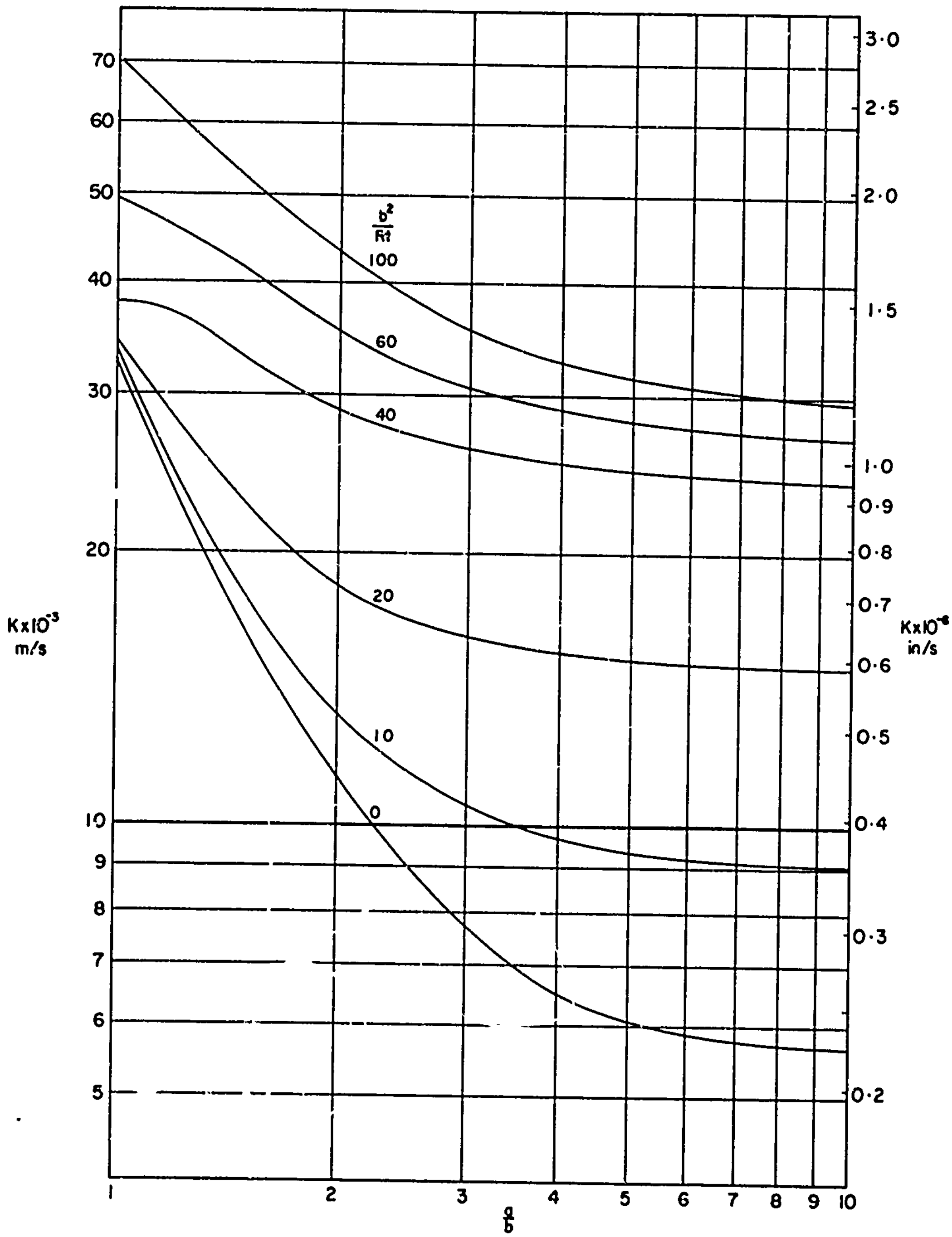


FIGURE 4.3. FREQUENCY PARAMETERS FOR SECOND SYMMETRIC-SYMMETRIC MODE (PLATES WITH ALL EDGES FIXED)

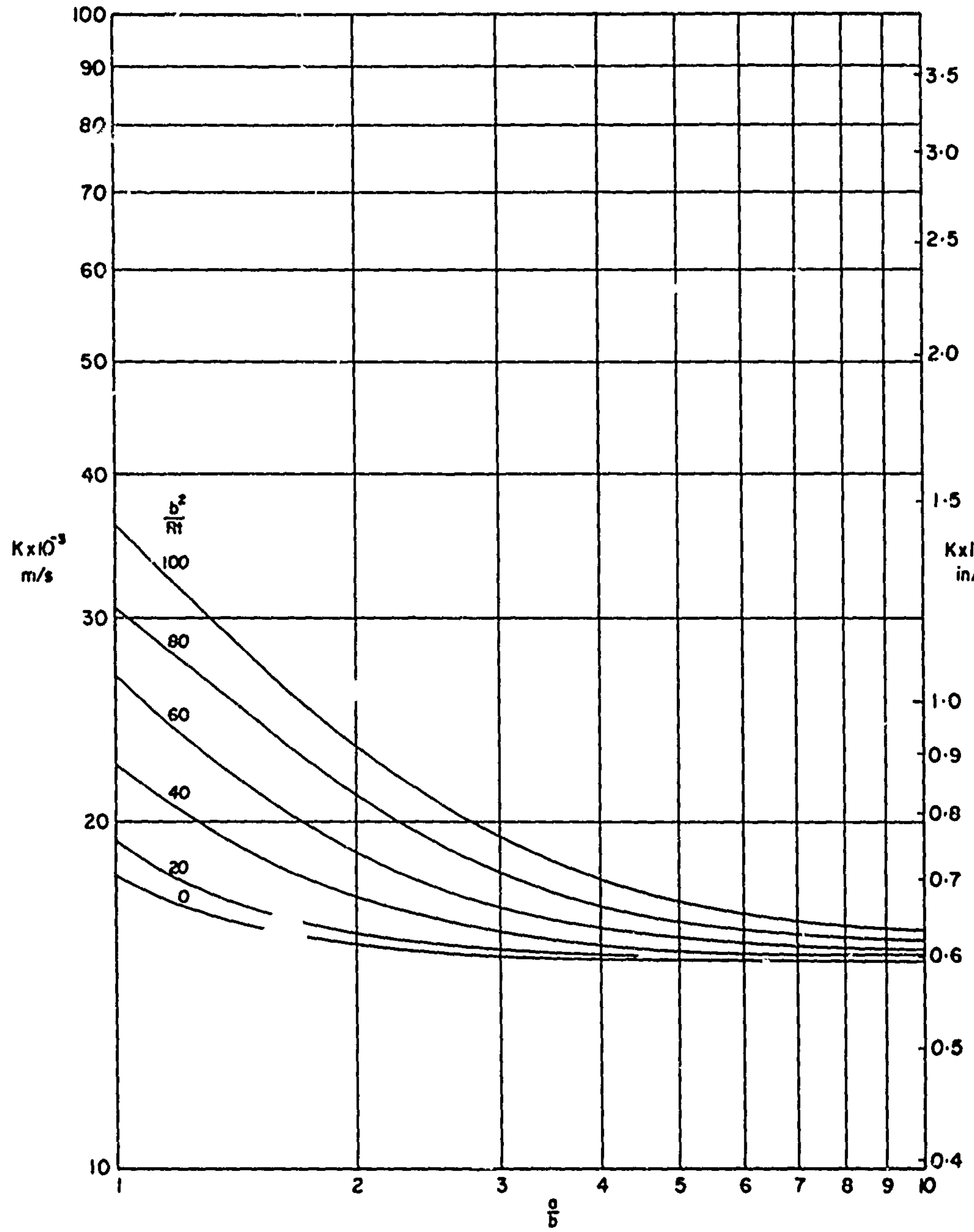


FIGURE 4.4. FREQUENCY PARAMETERS FOR FIRST SYMMETRIC-ANTISYMMETRIC MODE
(PLATES WITH ALL EDGES FIXED)

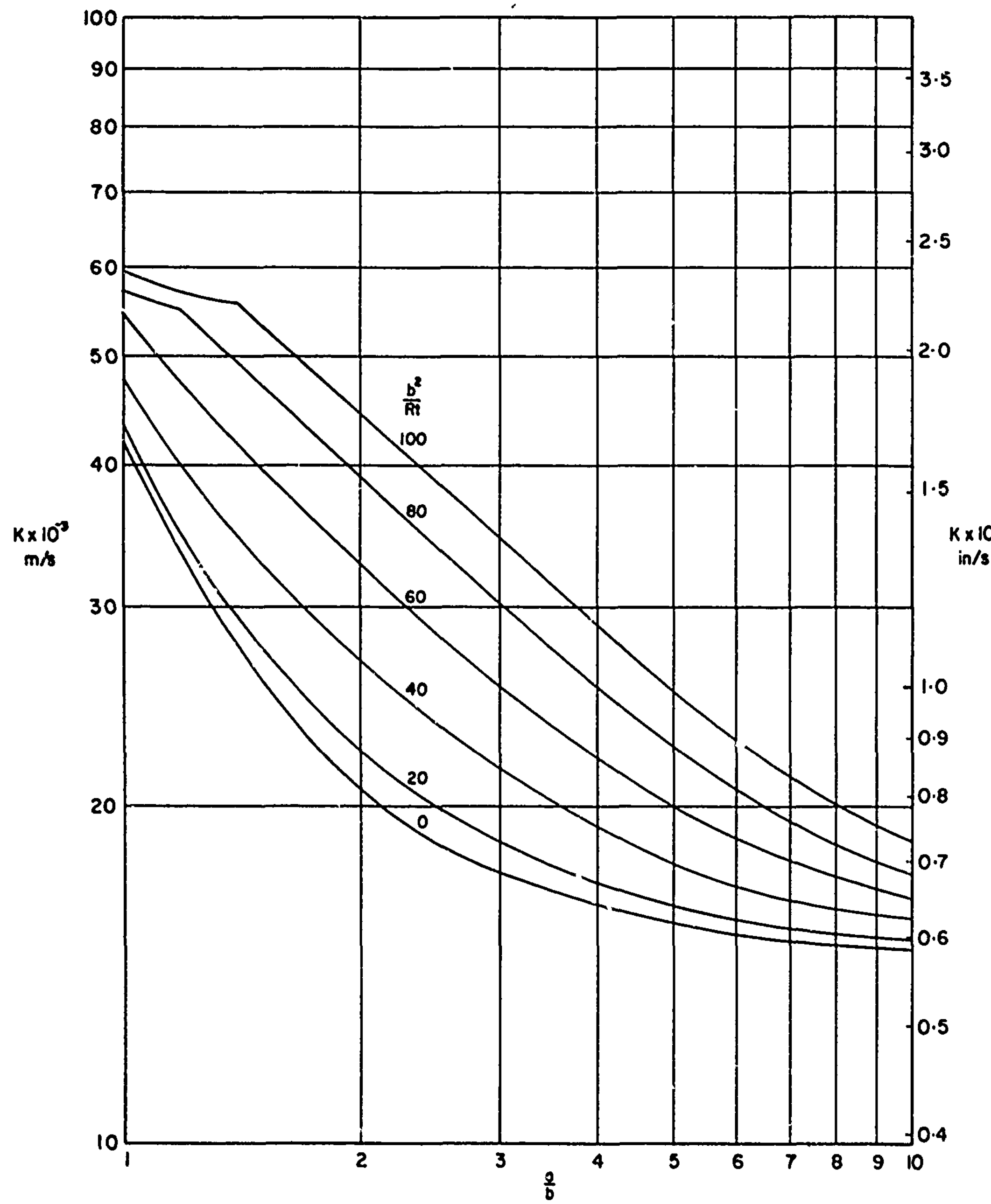


FIGURE 4.5 FREQUENCY PARAMETERS FOR SECOND SYMMETRIC-ANTISYMMETRIC MODE (PLATES WITH ALL EDGES FIXED)

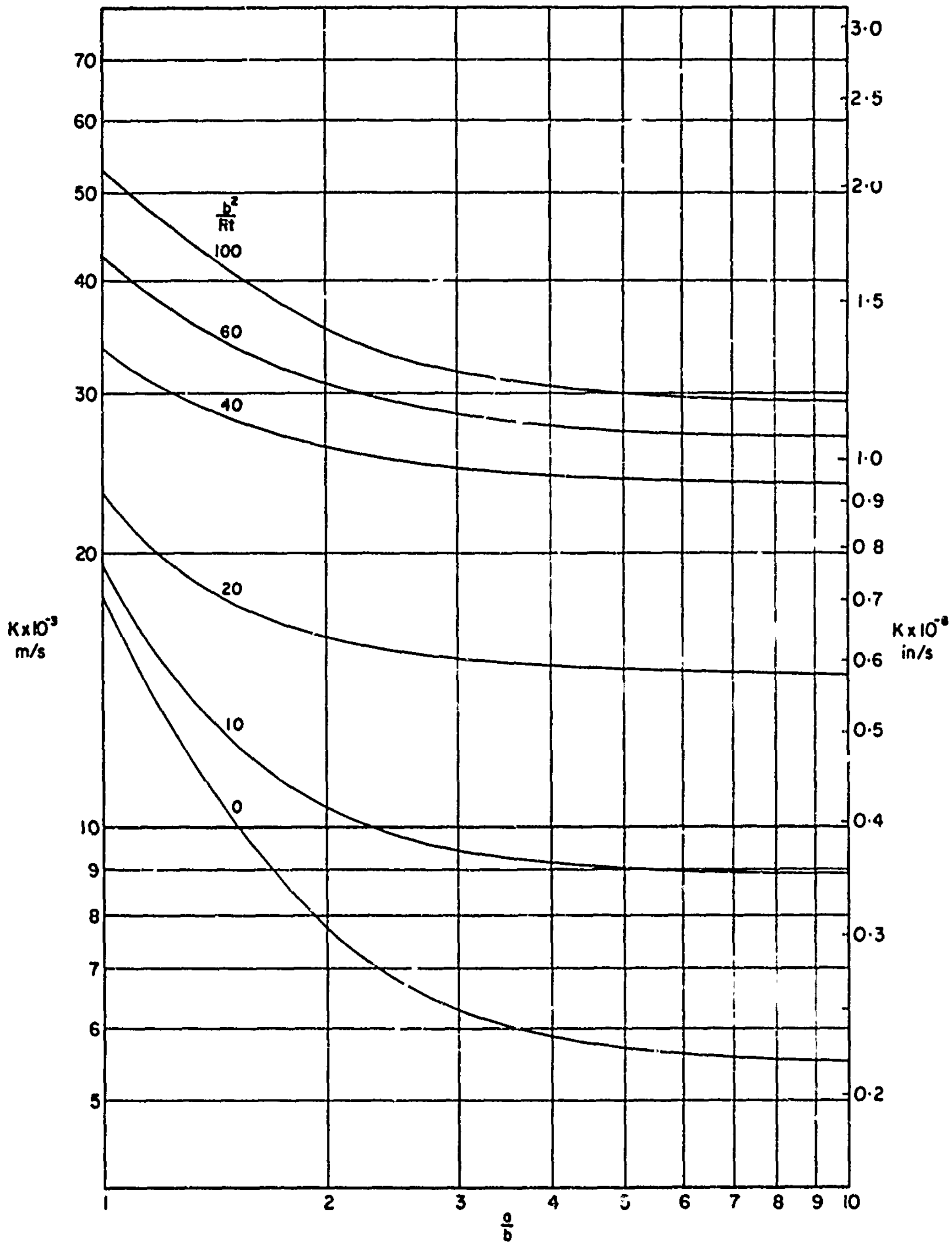


FIGURE 4.6. FREQUENCY PARAMETERS FOR FIRST ANTISYMMETRIC-SYMMETRIC MODE (PLATES WITH ALL EDGES FIXED)

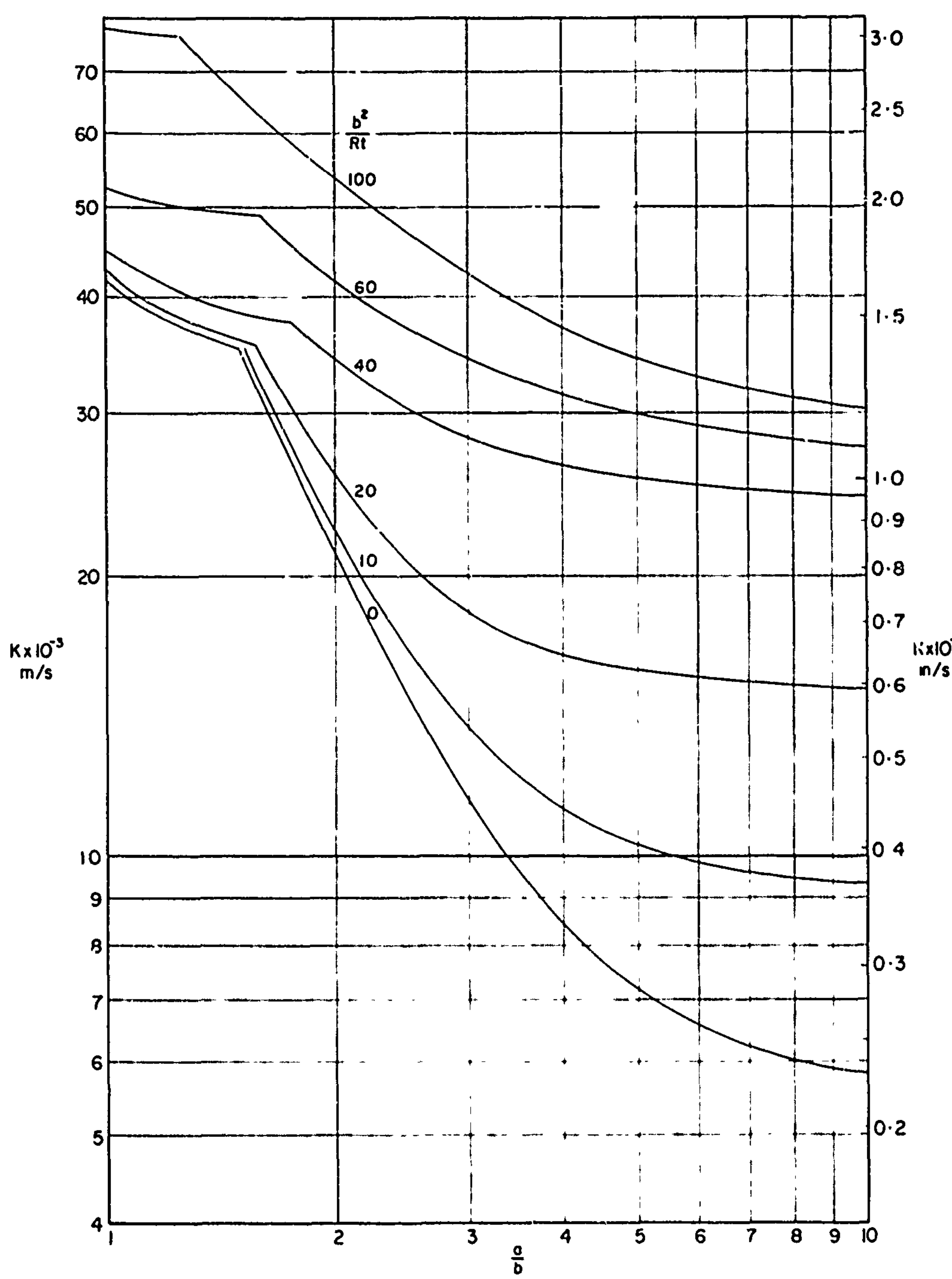


FIGURE 4 7. FREQUENCY PARAMETERS FOR SECOND ANTISYMMETRIC-SYMMETRIC MODE
(PLATES WITH ALL EDGES FIXED)

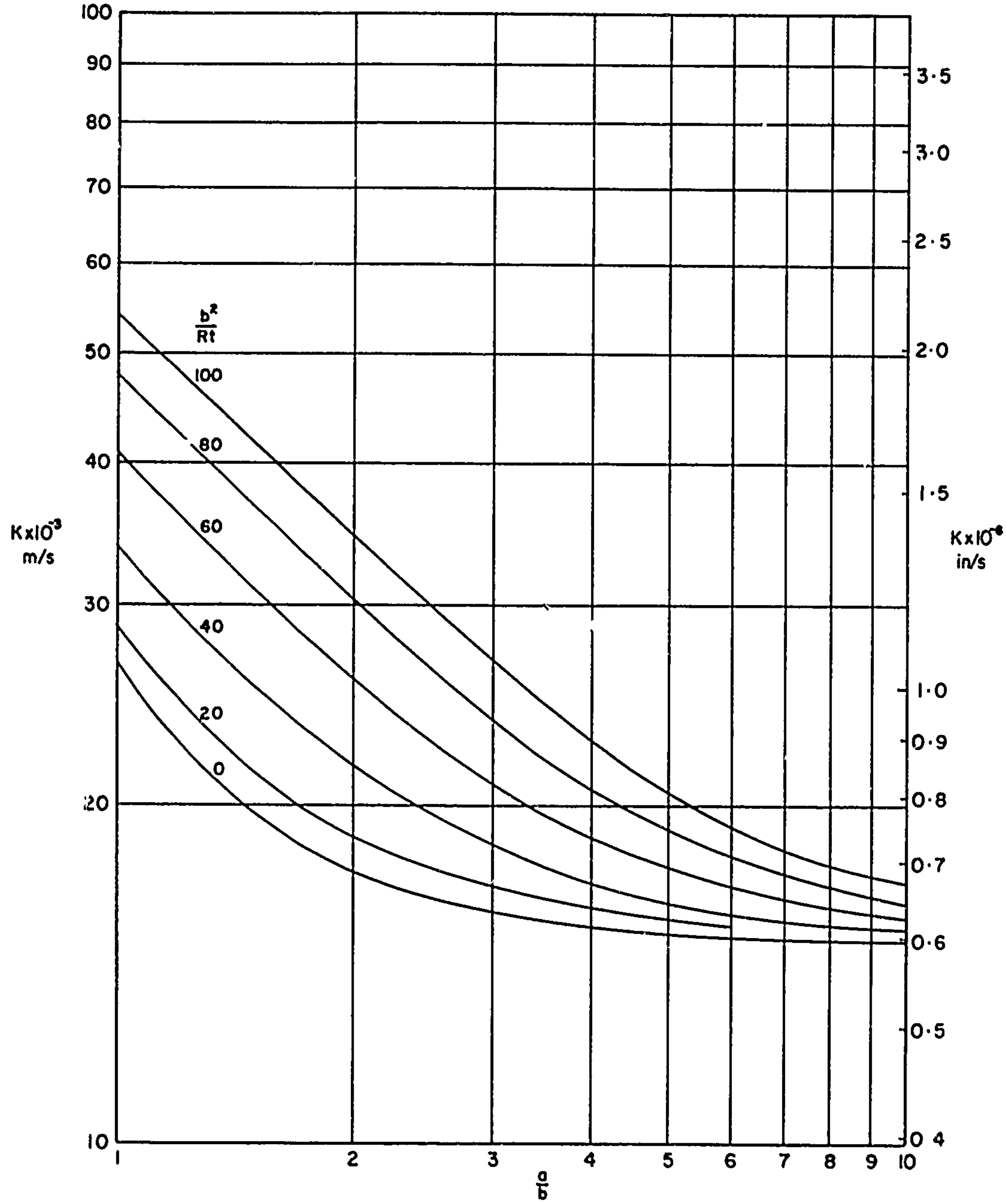
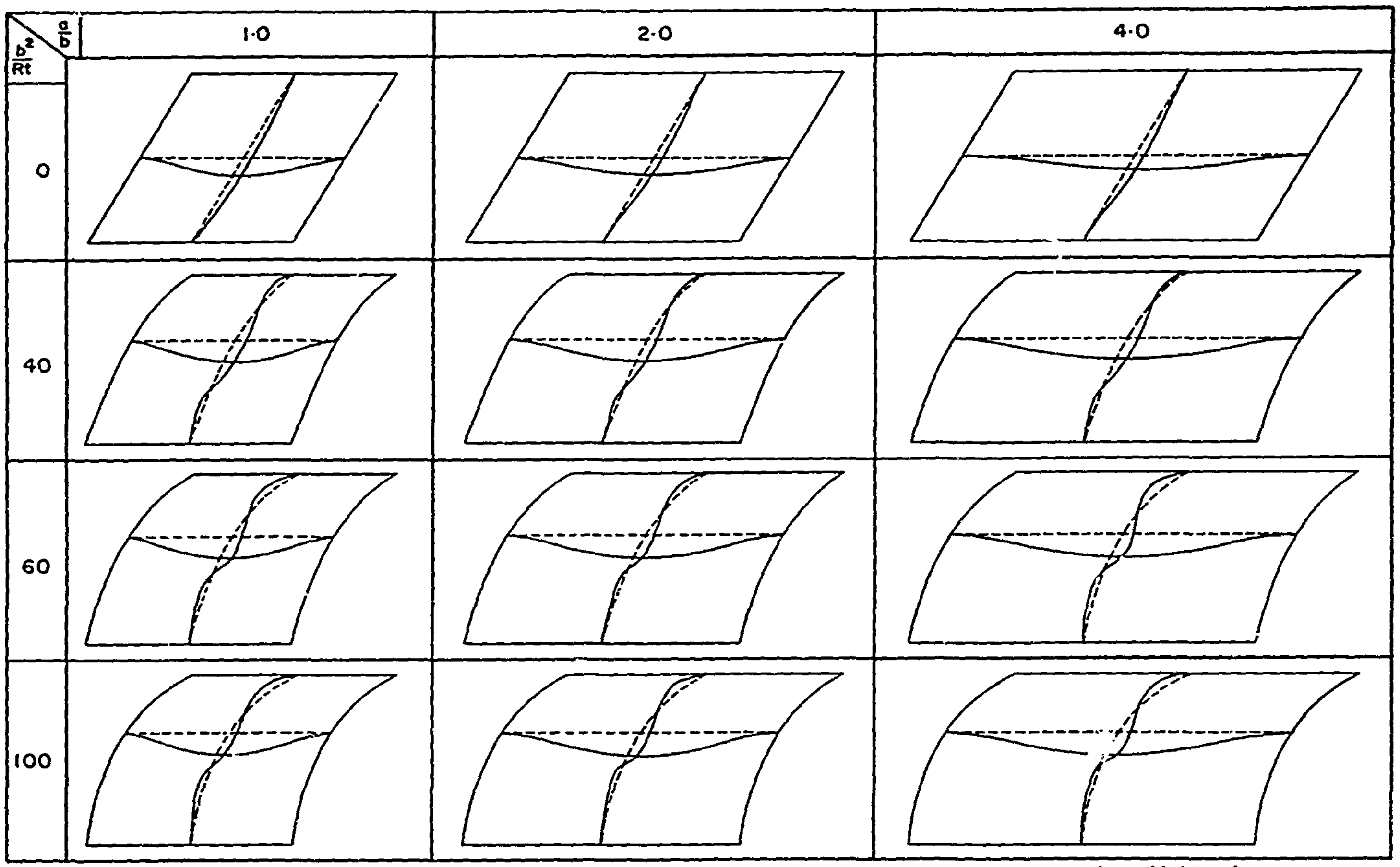


FIGURE 4.8. FREQUENCY PARAMETERS FOR FIRST ANTISYMMETRIC-ANTISYMMETRIC MODE
(PLATES WITH ALL EDGES FIXED)



t = 1·27 mm (0·050 in)

FIGURE 4.9. SHAPES FOR FIRST SYMMETRIC-SYMMETRIC MODE (PLATES WITH ALL EDGES FIXED)

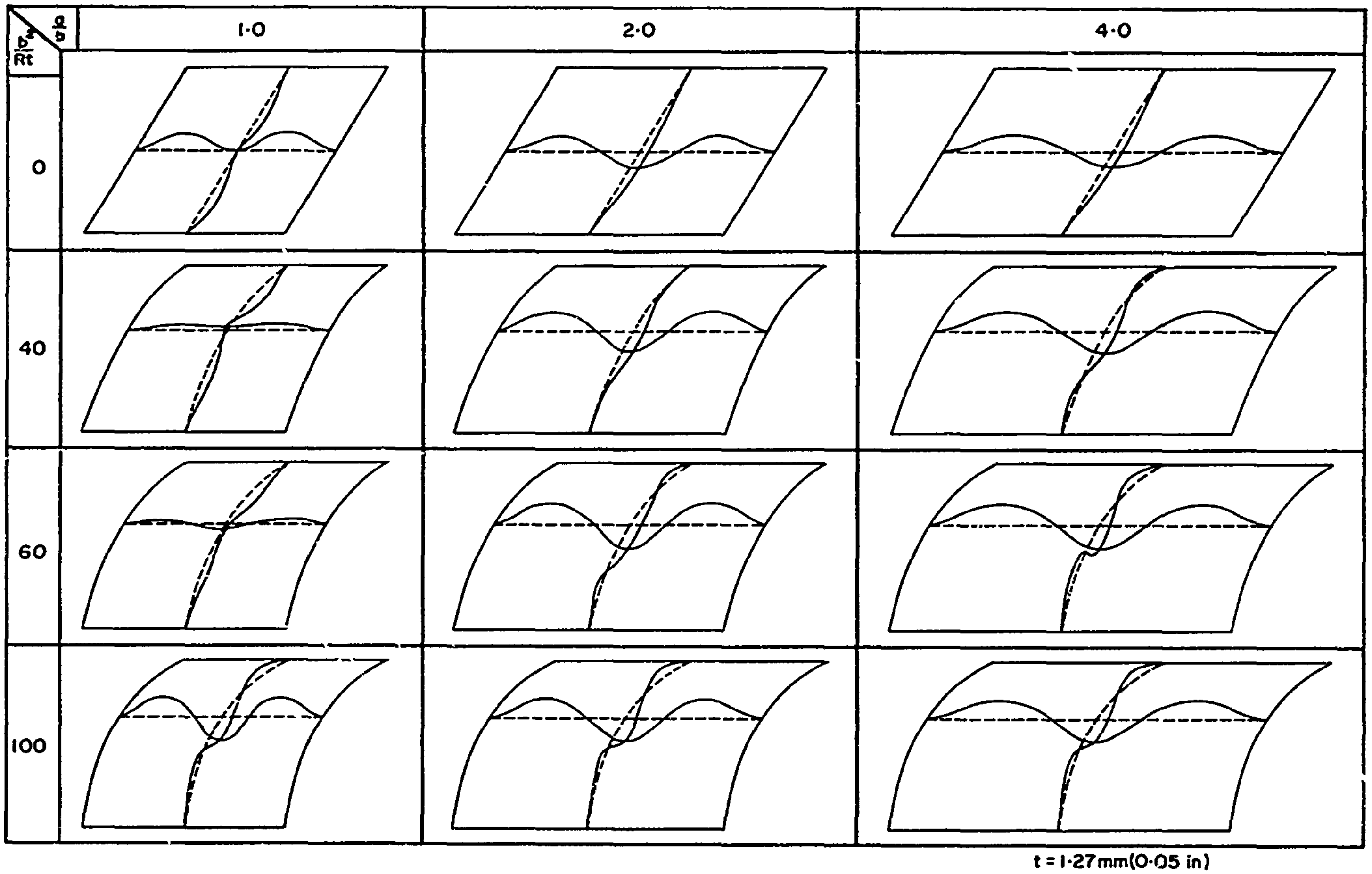


FIGURE 4.10. SHAPES FOR SECOND SYMMETRIC-SYMMETRIC MODE (PLATES WITH ALL EDGES FIXED)

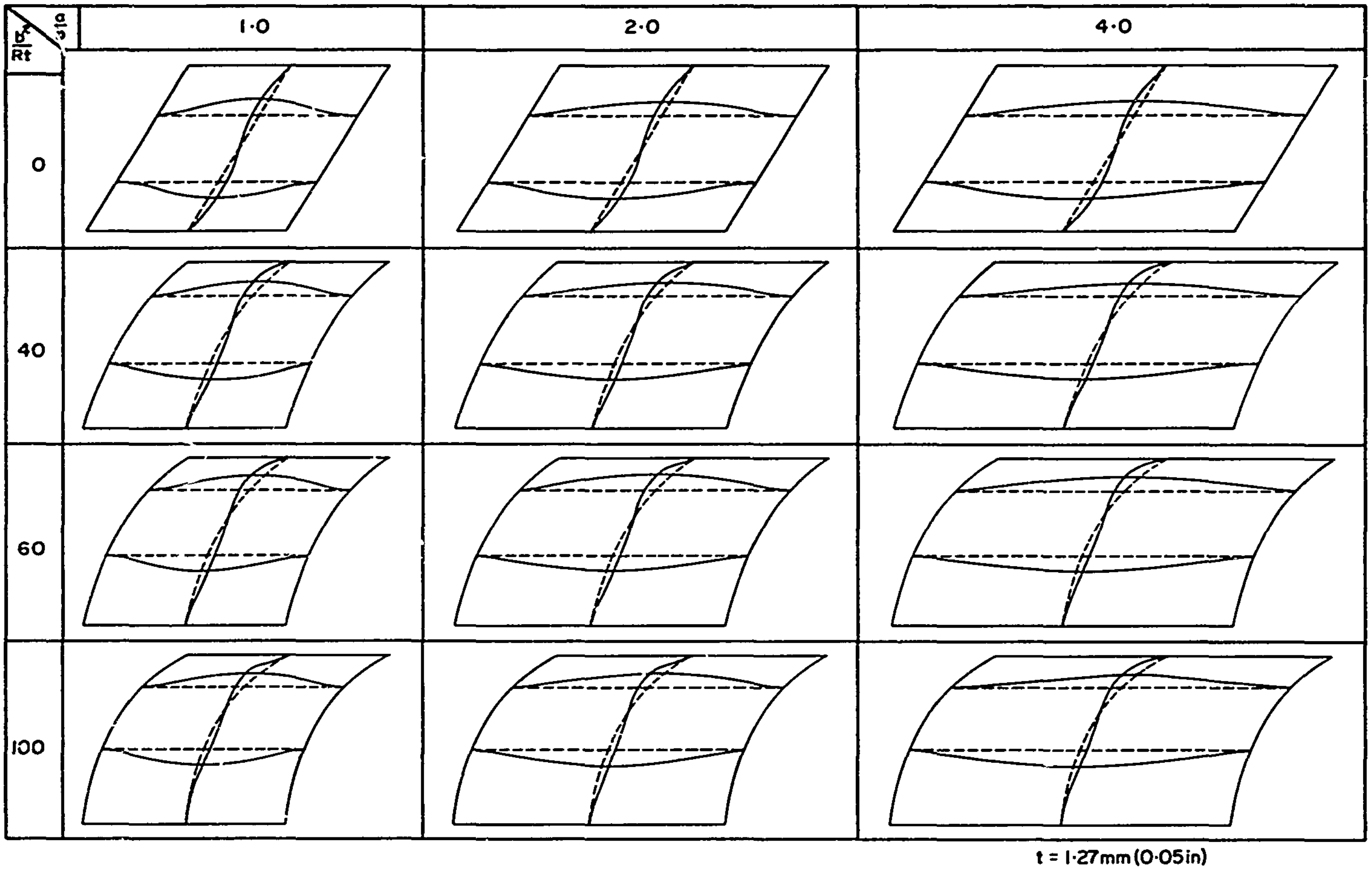


FIGURE 4.11 SHAPES FOR FIRST SYMMETRIC - ANTISYMMETRIC MODE (PLATES WITH ALL EDGES FIXED)

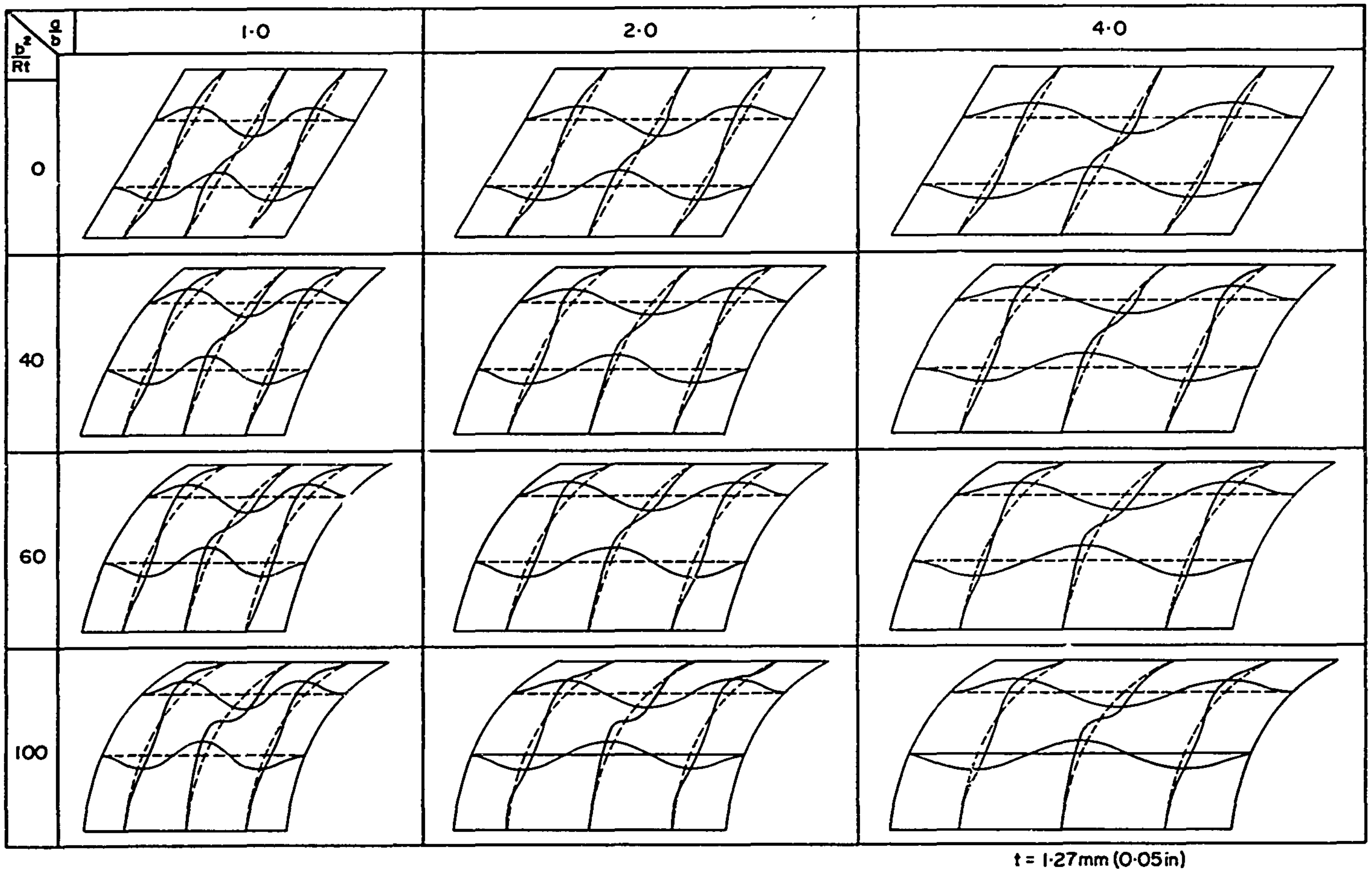


FIGURE 4.12 SHAPES FOR SECOND SYMMETRIC-ANTISYMMETRIC MODE (PLATES WITH ALL EDGES FIXED)

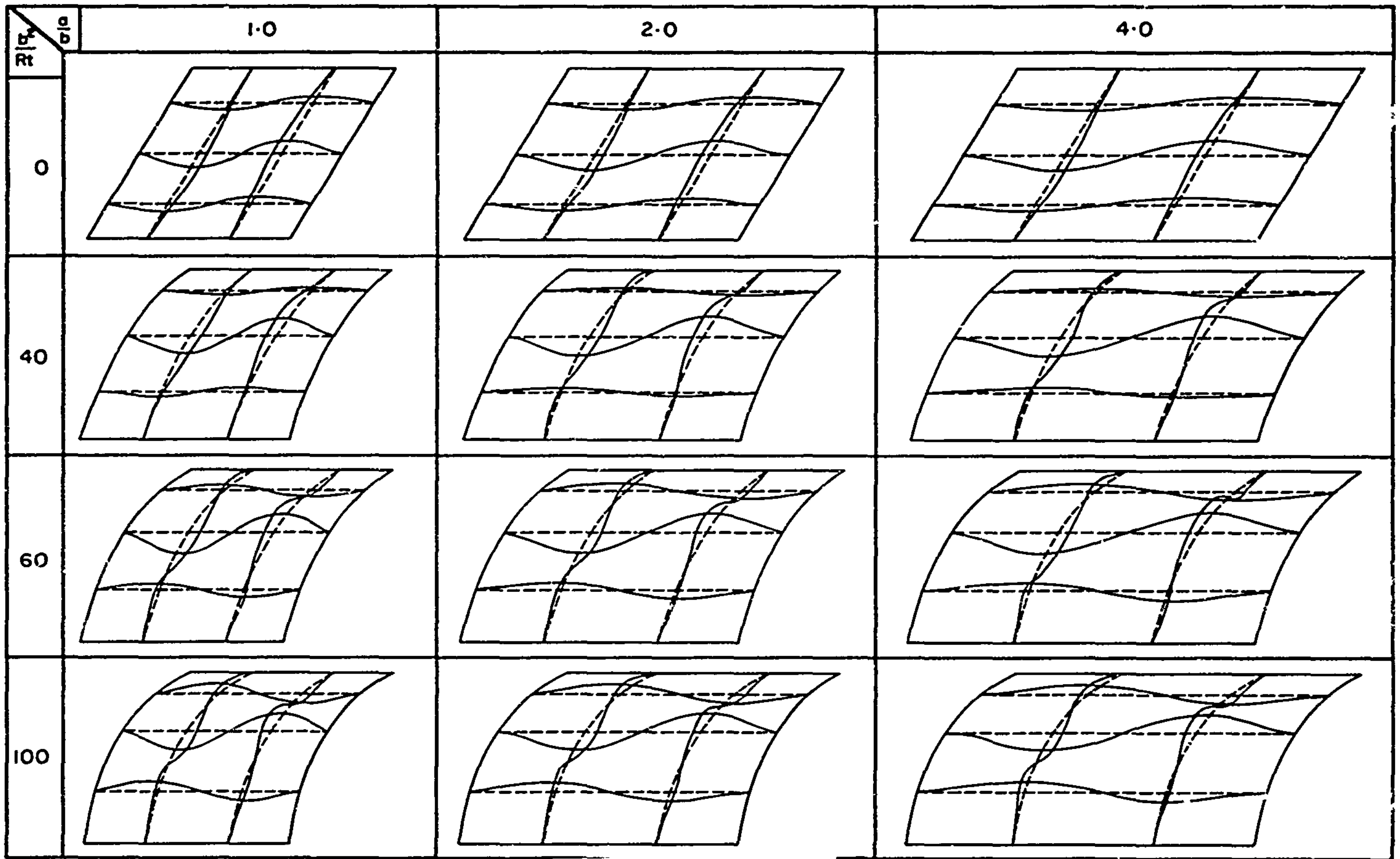
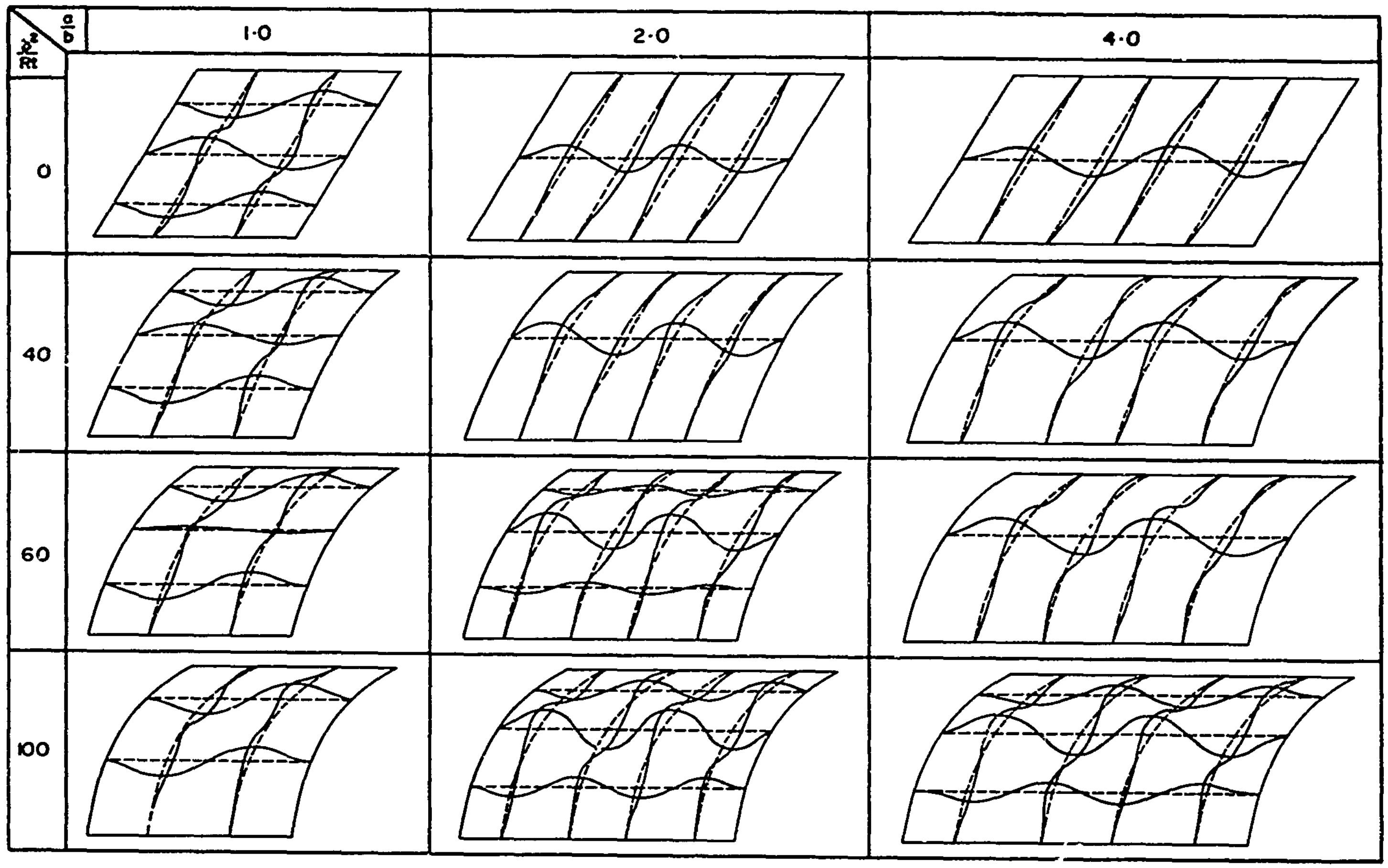
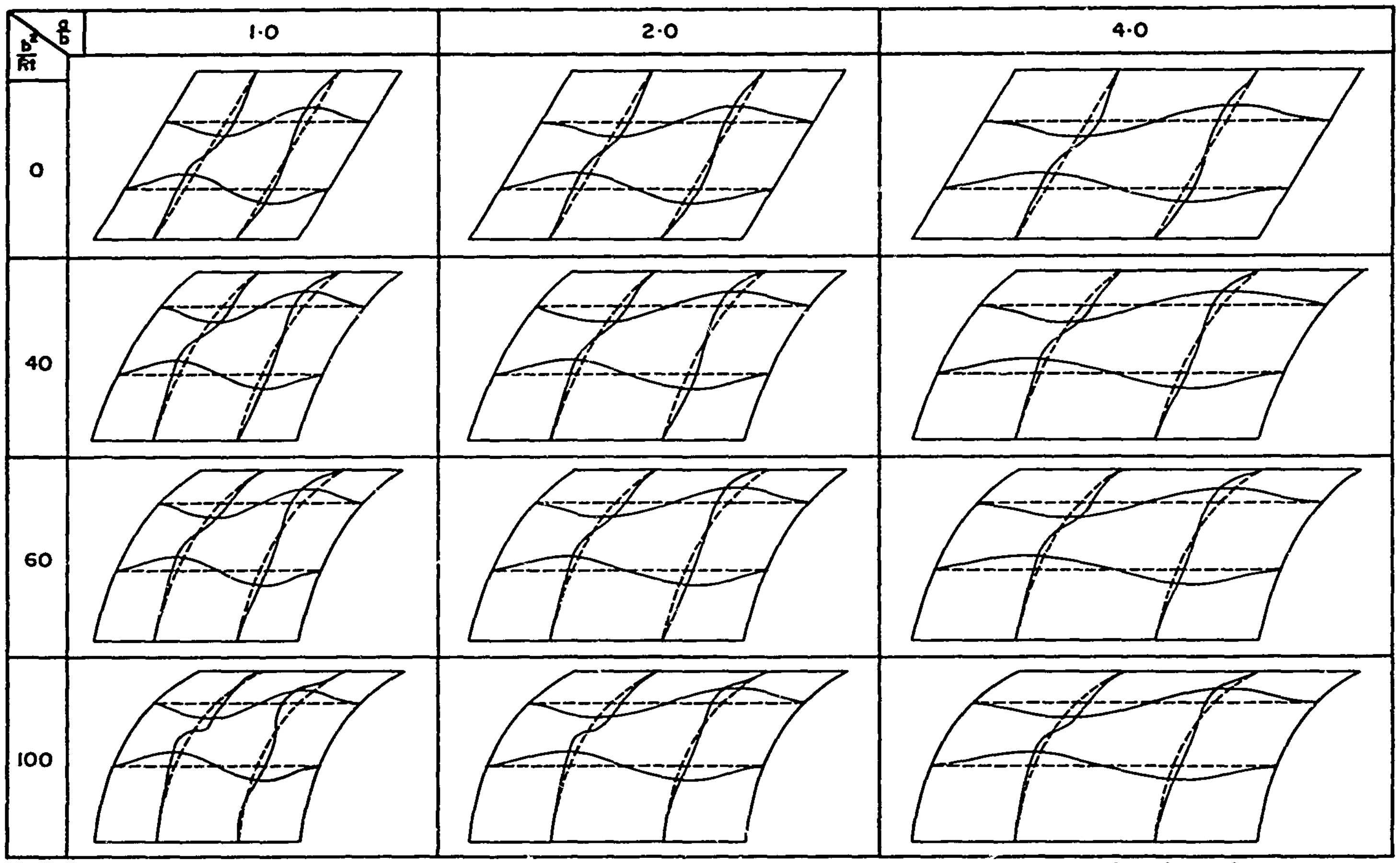


FIGURE 4 13 SHAPES FOR FIRST ANTISYMMETRIC-SYMMETRIC MODE (PLATES WITH ALL EDGES FIXED)



t = 1·27 mm (0·05)

FIGURE 4.14. SHAPES FOR SECOND ANTISYMMETRIC-SYMMETRIC MODE (PLATES WITH ALL EDGES FIXED)



t = 1·27 mm (0·05 in)

FIGURE 4 15. SHAPES FOR FIRST ANTISYMMETRIC-ANTISYMMETRIC MODE (PLATES WITH ALL EDGES FIXED)

Section 5

THE ESTIMATION OF R.M.S. STRESS IN STIFFENED SKIN PANELS SUBJECTED TO RANDOM ACOUSTIC LOADING

5.1 Notation

a	width of skin plate or frame pitch	m	in
b	flat panel width between stringers or curved panel arc length between stringers	m	in
E	Young's modulus of plate material	N/m^2	lbf/in^2
f	fundamental natural frequency of skin plate assuming all edges fixed	Hz	c/s
$G_p(f)$	spectral density of acoustic pressures at frequency f	$(N/m^2)^2/Hz$	$(lbf/in^2)^2/(c/s)$
K_δ	damping ratio correction factor		
$L_{ps}(f)$	spectrum level of acoustic pressure at frequency f	dB	dB
p	uniform static pressure on plate	N/m^2	lbf/in^2
p_{rms}	root mean square fluctuating pressure	N/m^2	lbf/in^2
R	radius of curvature of panel	m	in
S_o	ratio of stress at rivet line in assumed mode shape to applied uniform static pressure on plate		
S_{rms}	r.m.s. stress at rivet line due to acoustic loading	N/m^2	lbf/in^2
t	thickness of panel	m	in
V	velocity parameter for plate material+		
δ	damping ratio of mode of vibration of skin panel		
ρ	density of skin material	kg/m^3	*
σ	Poisson's ratio of plate material		

Both SI and British units are quoted but any coherent system of units may be used.

5.2 General Notes

This Section gives a method of estimating the r.m.s. stress in rectangular skin panels subject to random acoustic loading. The reference stress is the rivet line stress; stresses at other positions on the plate may be found by applying a suitable factor to the reference stress.

The r.m.s. stress for a stiffened panel subjected to random acoustic loading on one side is given approximately by the expression

$$S_{rms} = \left[\frac{\pi}{4\delta} f\, G_p(f)\right]^{1/2} K_\delta S_o \,. \qquad (5.1)$$

+ The velocity parameter is defined in SI units by $V = (E/\rho)^{\frac{1}{2}}/5080$ or in British units by $V = (E/\rho)^{\frac{1}{2}}/200\,000$. V is approximately unity for all common structural metallic materials.

* A density value expressed in British units as pounds per cubic inch has to be divided by 386.4 before it can be used in the formula for V given here. (A force of 1 lbf acting on a mass of 1 lb produces an acceleration of 386.4 in/s^2.)

In the case of a control surface or similar structure where two skins are coupled by ribs and both vibrate in response to a loading largely on one side, the stress level is about 1/3 of the value given by Equation (5.1). However, where the acoustic loading is of similar intensity on both sides of a box structure the stress level is about 1/2 of the value given by Equation (5.1).

In deriving Equation (5.1) it has been assumed that the predominant form of skin vibration is one in which individual plates within the stiffened panel vibrate independently in the assumed mode which, for this calculation, has been taken to be the fundamental fixed-edge mode. This restricts the use of this Section to skin-stringer panels where the bending stiffness of the stringers is sufficiently high to approximate to a fixed-edge condition for individual plates. It is not applicable to stiffened panels where individual plates are much stiffer and the supporting stiffeners relatively more flexible than for conventional skin-stringer panels.

The pressure is assumed to be uniform and in phase over the whole of each individual plate and the spectrum level of the acoustic pressure is taken to be constant over the range of frequencies close to the fundamental natural frequency of the panel.

The r.m.s. stresses given are the nominal unfactored rivet line values and in using these stresses to calculate a fatigue life care must be taken to ensure due account is taken of any geometric stress concentrations in the region of the rivet line.

This Section may be used for etched skins, or skins with bonded reinforcing, by first obtaining the nominal rivet line r.m.s. stress for the panel, assuming it to be of uniform thickness, and then applying a correction with the aid of Reference 5.6.14.

For conventional structures without special damping treatment, f may be taken as the undamped natural frequency.

5.3 Notes on the Figures

Figures 5.1 and 5.2 give nomographs for S_{rms}. The nomograph are entered at a value of $L_{ps}(f)$, each quadrant being used in turn in the direction indicated through ranges of a/b, b/t, b^2/Rt and t/V. Figure 5.2 is an extension of the stress range of Figure 5.1. Figures 5.1 and 5.2 are drawn for a value of $\delta = 0.017$; S_{rms} for other values of δ is calculated using correction factor K_δ which is plotted against δ in Figure 5.3

The r.m.s. stress given is the skin surface stress across the rivet line at the mid position of the longer side of the plate. This is the position of highest nominal stress when the plates vibrate in their fundamental fixed-edge mode. No allowance is made for stress concentration factor due to rivet holes etc.; this is expected to be negligible at distances greater than 2.5 times the rivet diameter from the centre line of the rivet along the rivet line.

In the derivation of the nomographs it is assumed that the plate bending stress is within the linear region where it is directly proportional to the normal pressure, that is p/E less than about $20(t/b)^4$.

The higher range of Figure 5.2 is for use in estimating stresses in titanium and high strength steel panels. However, high values of stresses obtained from this Figure should be treated with caution as no extensive measured data are at present available to check the validity of the simple response theory over the higher stress range.

5.4 Calculation Procedure

5.4.1 The procedure for estimating S_{rms} in a general case is as follows.

(i) Estimate the fundamental natural frequency of the panel. For flat plates use Reference 5.6.13 or Section 4 of this AGARDograph, or evaluate the highest frequency of the first group of natural frequencies using Section 3 of this AGARDograph. For curved plates calculate the natural frequency using Section 4 of this AGARDograph.

(ii) Obtain the value of spectrum level of acoustic pressure $L_{ps}(f)$ at the calculated frequency. If only the band pressure level is known, it is first corrected to pressure spectrum level (unit bandwidth) using Reference 5.6.11. The reference pressure for sound pressure level is $20 \mu N/m^2$.

(iii) Calculate the parameters a/b, b/t, b^2/Rt and t/V and read the value of S_{rms} from Figures 5.1 or 5.2.

(iv) For values of δ other than 0.017, factor the estimated value of S_{rms} by K_δ obtained from Figure 5.3. The value of $\delta = 0.017$ is typical of aircraft structures without special damping treatment.

5.4.2 Within the nomograph the spectrum sound pressure level is converted into the spectral density of acoustic pressure. The spectrum sound pressure level is converted into the root mean square fluctuating pressure in units of $(N/m^2)/Hz$ (see expression below or Reference 5.6.12) and then squared giving a value in units of $(N/m^2)^2/Hz^2$. Since unit bandwidth is used this is numerically equal to the spectral density of acoustic pressure $G_p(f)$ in units of $(N/m^2)^2/Hz$.

$$L_{ps}(f) = 20(\log_{10} p_{rms} + 4.70) .$$

If $L_{ps}(f)$ is required in British units of $(lbf/in^2)^2/(c/s)$ it is given by

$$L_{ps}(f) = 20(\log_{10} p_{rms} + 8.54) .$$

5.5 Comparison with Measured Data

Figure 5.4 shows a comparison of estimated and measured stresses in flat or curved plates. Figure 5.5. shows a comparison of estimated and measured stresses in control surfaces and other coupled skin box type structures. For these estimated values the factors given in Section 5.2 were applied to the stresses given by Equation (5.1). In estimating r.m.s. stresses to compare with measured values, δ has been assumed to be 0.017.

5.6 Derivation and References

Derivation

5.6.1	Schjelderup, H.C.	Structural acoustic proof testing. Aircr. Engng, Vol.31, No.386, pp.297-303, October 1959.
5.6.2	Clarkson, B.L.	The design of structures to resist jet noise fatigue. J. R. aeronaut. Soc., Vol.66, No.622, pp.603-616, October 1962.
5.6.3	Wagner, J.G.	Caravelle acoustical fatigue. Proceedings of ICAF Symposium on Fatigue of Aircraft Structures, Paris, May 1961, pp.321-343, Pergamon Press, 1963.
5.6.4	Eaton, D.C.G.	Unpublished work at British Aircraft Corporation Ltd, 1965.
5.6.5	-	Unpublished work at the Boeing Company Ltd, 1967.
5.6.6	Ballentine, J.R. et al.	Refinement of sonic fatigue structural design criteria. Air Force Flight Dynamics Lab., Ohio, tech. Rep. AFFDL-TR-67-156, November 1967.
5.6.7	Clarkson B.L.	Stresses in skin panels subjected to random acoustic loading. J. R. aeronaut. Soc., Vol.72, No.695, pp.1000-1010, November 1968.

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5.6.8	Timoshenko, S.P. Woinowsky-Krieger, S.	Theory of plates and shells. Second edition. McGraw Hill, New York, 1959.
5.6.9	Lin, Y.K.	Free vibrations of continuous skin stringer panels. J. appl. Mech., Vol.27, pp.669-676, December 1960.
5.6.10	Lin, Y.K.	Stresses in continuous skin-stiffener panels under random loading. J. Aero/Space Sci., Vol.29, No.1, pp.67-75, January 1962.

5.6.11 - Bandwidth correction.
Engineering Sciences Data Item No. 66016, February 1966.

5.6.12 - The relation between sound pressure level and r.m.s. fluctuating pressure.
Engineering Sciences Data Item No. 66018, February 1966.

5.6.13 - Natural frequencies of uniform flat plates.
Engineering Sciences Data Item No. 66019, February 1966.

5.6.14 - The effect of edge reinforcement on the stresses in skin panels under uniform pressure.
Engineering Sciences Data Item No. 67029, May 1967.

5.7 Examples

5.7.1 It is required to estimate the rivet line r.m.s. stress in a stiffened panel subjected to jet noise on one side. The variation of found pressure level over a range of frequencies is given in the table, sound pressure levels being 1/3 octave band levels.

Sound pressure level(dB)	130	145	150	148	143	130
Frequency (Hz)	150	200	300	500	700	1000

The panel is made up from uniform plates having the following dimensions and properties:

$$a = 210 \text{ mm}, \quad b = 140 \text{ mm}, \quad t = 1.2 \text{ mm}, \quad R = 1500 \text{ mm},$$

$$E = 70\,000 \text{ MN/m}^2, \quad \rho = 2770 \text{ kg/m}^3, \quad \delta = 0.020 .$$

Firstly

$$\frac{a}{b} = \frac{210}{140} = 1.5 ,$$

$$\frac{b}{t} = \frac{140}{1.2} = 116.7 ,$$

$$\frac{b^2}{Rt} = \frac{140^2}{1500 \times 1.2} = 10.9$$

$$V = \sqrt{\frac{70\,000 \times 10^6}{2770}} \times \frac{1}{5080} = 0.99$$

and

$$\frac{t}{V} = \frac{1.2}{0.99} = 1.21 \text{ mm}.$$

From Section 4 of this AGARDograph the fundamental natural frequency of the plates with all edges fixed is obtained.

$$f = 0.99 \times 10\,200 \times \frac{1.2 \times 1000}{140^2} = 618 \text{ Hz}.$$

By interpolation from the table the 1/3 octave band pressure level at 618 Hz is 145.7 dB.

From Reference 5.6.11

$$L_{ps}(f) = 145.7 - 21.7 = 124 \text{ dB}.$$

From Figure 5.1, entering the nomograph at 124 dB, $S_{rms} = 14.0$ MN/m^2 for $\delta = 0.017$.

From Figure 5.3 for $\delta = 0.020$, $K_\delta = 0.922$. Hence the rivet line r.m.s. stress $= 14.0 \times 0.922 = 12.9$ MN/m^2.

5.7.2 If a doubler is now bonded to the skin at the rivet line the new stresses may be found using Reference 5.6.14. Additional definitions in this Reference are:

b_r	width of reinforcement	m	in
f_o	nominal stress at rivet line in unreinforced panel	N/m^2	lbf/in^2
f_1	nominal stress at rivet line in reinforced panel	N/m^2	lbf/in^2
f_2	nominal stress at edge of reinforcement	N/m^2	lbf/in^2
t_r	total thickness of panel and reinforcement	m	in

In this example f_o is the previously calculated value of S_{rms} .

The doubler is the same material as the plate and has the following dimensions:

$$b_r = 21 \text{ mm} \quad \text{and} \quad t_r = 1.8 \text{ mm}.$$

Firstly

$$\frac{b_r}{b} = \frac{21}{140} = 0.15$$

and

$$\frac{t_r}{t} = \frac{1.8}{1.2} = 1.5.$$

From Reference 5.6.14

$$\frac{f_1}{f_o} = 0.485 \quad \text{and} \quad \frac{f_2}{f_o} = 0.677.$$

Hence the rivet line r.m.s. stress with the doubler

$$= 12.9 \times 0.485 = 6.3 \text{ MN/m}^2$$

and the r.m.s. stress at the edge of the doubler

$$= 12.9 \times 0.677 = 8.7 \text{ MN/m}^2.$$

It should be noted that Reference 5.6.14 is strictly applicable to panels of $a/b > 2$. Within the accuracy of the simple theory for stress response to acoustic loading it may be used for lower values of a/b .

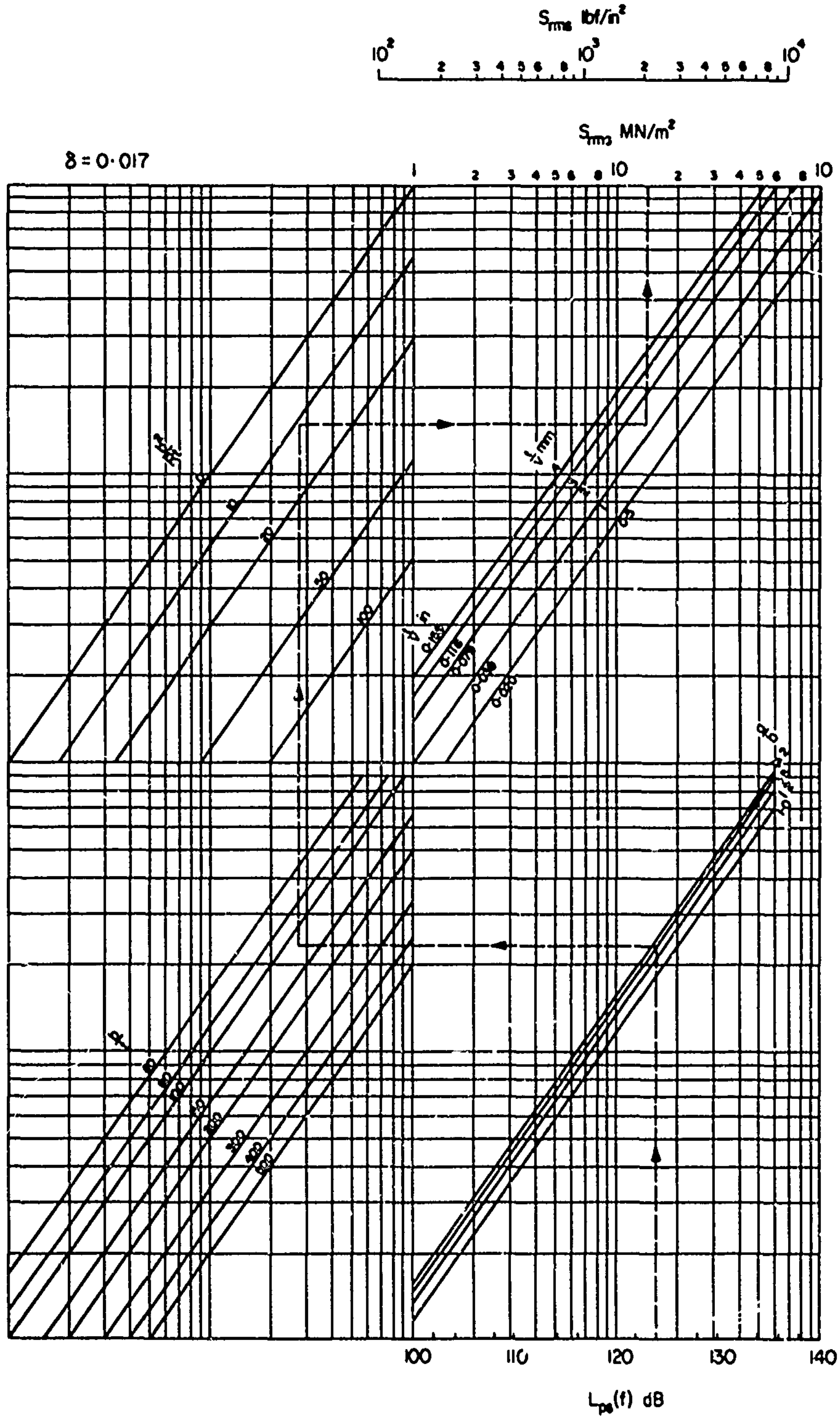

FIGURE 5.1. LOW RANGE STRESS NOMOGRAPH

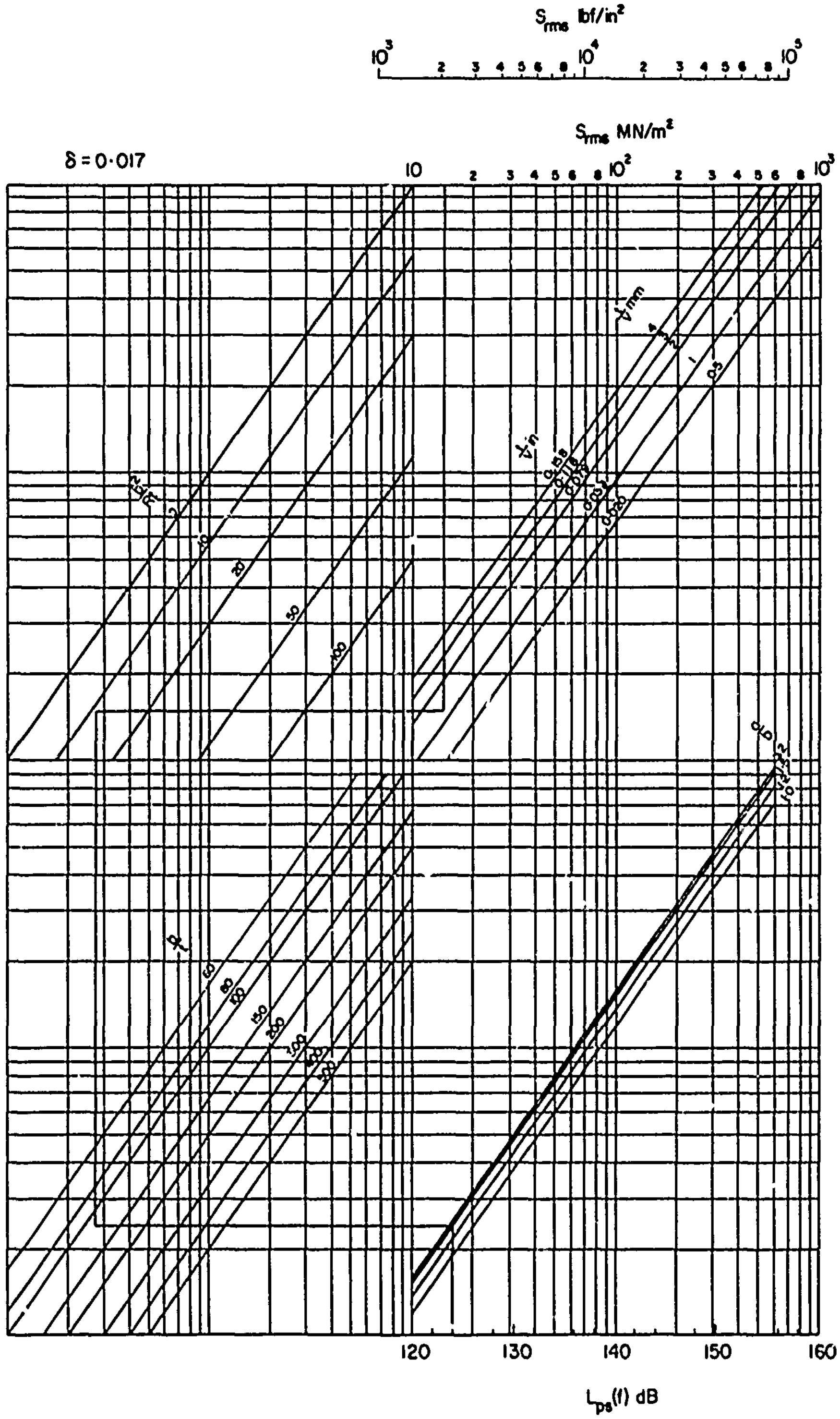

FIGURE 5.2. HIGH RANGE STRESS NOMOGRAPH

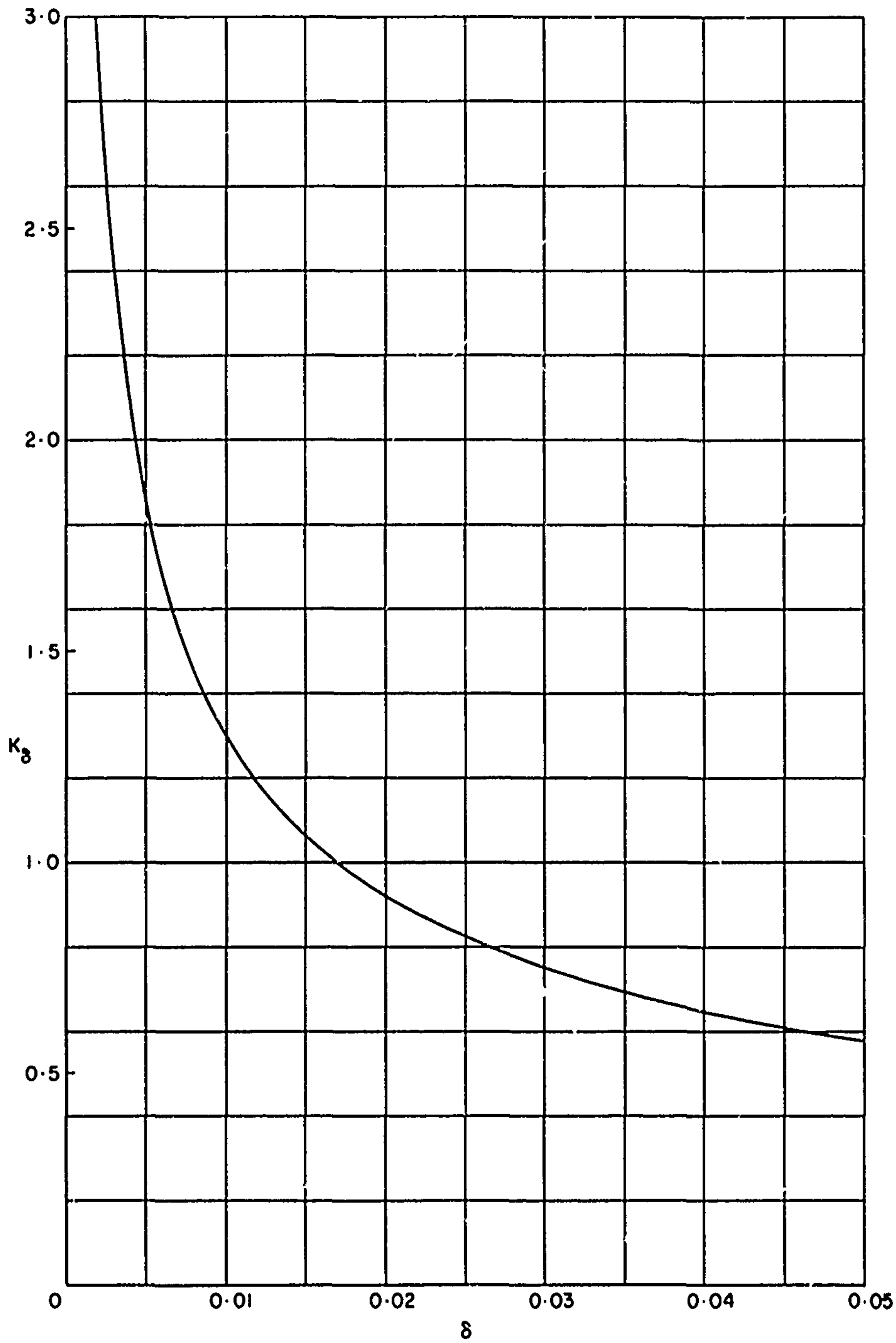

FIGURE 5.3 DAMPING RATIO CORRECTION FACTOR

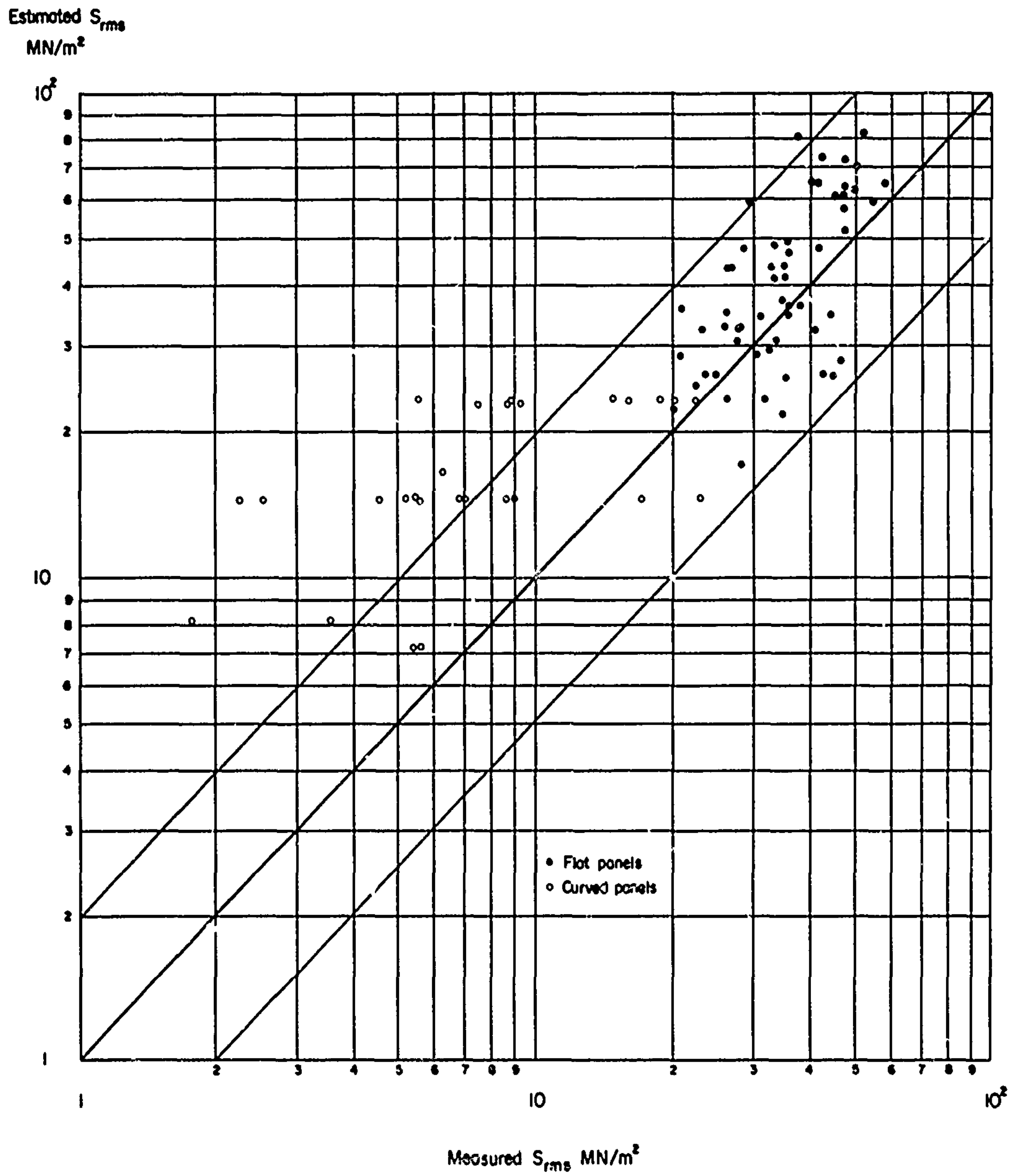

FIGURE 5.4. COMPARISON OF ESTIMATED AND MEASURED STRESSES IN FLAT AND CURVED PANELS

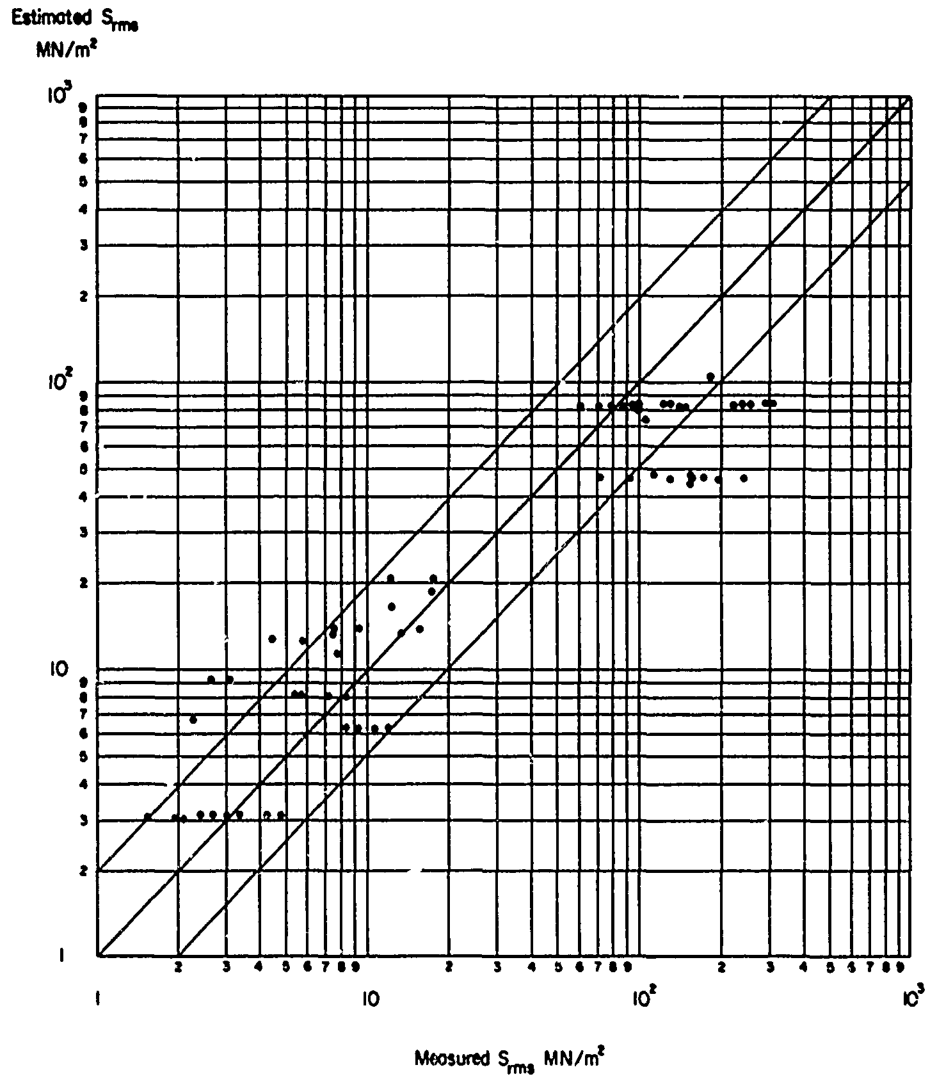

FIGURE 5.5. COMPARISON OF ESTIMATED AND MEASURED STRESSES IN TAILPLANES AND CONTROL SURFACES